An Energetic Perspective on the Tropical Atmosphere and its Response to Climate Warming

by

Margaret Louise Duffy

Submitted to the Department of Earth, Atmospheric, and Planetary Sciences
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Abstract

This thesis evaluates the dynamics of the tropical atmosphere and its response to warming using energetic approaches. We focus on three features of the atmosphere over tropical oceans: the response of precipitation to warming, the gross moist stability (GMS), and the response of the Pacific Walker circulation (WC) to warming. There have been a number of mechanisms proposed to explain the response of precipitation to warming. The “wet-get-wetter” mechanism describes an amplification of the pattern of precipitation in a moister atmosphere, and the “warmer-get-wetter” mechanism describes enhanced upward motion and precipitation in regions where the increase in sea surface temperature (SST) exceeds its tropical-mean increase. Studies of the current climate have shown that surface convergence (SC) over the tropical oceans is largely driven by horizontal gradients of low-level temperature. Chapter 2 finds that a “Laplacian-of-warming” mechanism is of comparable importance to wet get wetter and warmer get wetter for the response of precipitation to climate change over tropical oceans. The GMS quantifies the energy import or export of a circulation but, despite its importance, is a difficult quantity to understand and to observe. Chapter 3 approximates the vertical GMS using SST and the Laplacian of SST and finds that the approximation works well in the mean and seasonal cycle. There is uncertainty about the sign and magnitude of the response of the WC to warming. Chapter 4 finds a strong relationship between GMS response and WC strength response across a hierarchy of GCMs. Further, Chapter 4 finds that WC strength and GMS responses are sensitive to the degree of parameterized convective entrainment, but that the spread in GMS responses due to differing entrainment rates is smaller than the spread in GMS response across CMIP5 models. Taken together, this thesis progresses our understanding of the tropical circulation and precipitation pattern and its response to warming.

Thesis Supervisor: Paul O’Gorman
Title: Professor of Atmospheric Science
Dedication

This thesis is dedicated Angeline C. Duffy. Nanny, as I called her, was my paternal grandmother and was a pioneering woman in climate science as a research assistant in John Imbrie’s group at Brown University. She was involved in seminal work relating ice ages to Earth’s orbital parameters (Imbrie et al., 1984, 1992, 1993; Matthews et al., 1997).

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Chapter 1

Introduction

1.1 Overview

This thesis examines the dynamics of the tropical atmosphere with an emphasis on its response to anthropogenic climate change. In particular, the thesis examines the large-scale circulation and precipitation over oceans, which are related to one another. The tropical atmosphere includes a meridional circulation, the Hadley circulation, which transports energy across the equator from the summer to winter hemisphere. Simultaneously, the Walker circulation (WC) transports energy zonally over each ocean basin. Here we focus on the Pacific Walker circulation which transports energy from the warmer west Pacific to the cooler east Pacific. The efficiency of the circulations in transporting energy can be measure by the gross moist stability (GMS). The ascent regions of these circulations tend to be wet and the descent regions tend to be dry. Most prominently, the ascent region of the Hadley circulation is the Intertropical Convergence Zone (ITCZ), a cloudy band of strong rainfall. This thesis makes progress on understanding the dynamics of the tropical atmospheric circulation, especially the Walker circulation; the mechanics of GMS; and the response of climatological precipitation to warming.

The relationship between the large-scale circulation, precipitation, and the response to warming is easily seen when considering the following approximate equa-
where \( P \) is precipitation, \( M \) is convective mass flux, and \( q \) is specific humidity. Equation 1.1 reveals that, for a given humidity, precipitation increases with increasing mass flux. In the response to warming, \( q \) increases at a rate of about 7% K\(^{-1}\), consistent with the Clausius-Clapeyron relationship. However, in model simulations precipitation increases at a smaller rate than specific humidity, about 2% K\(^{-1}\). This indicates that convective mass fluxes must, on average, weaken with warming (Held and Soden, 2006; Vecchi and Soden, 2007; Chadwick et al., 2013). However, the implications for the large-scale circulation are less clear (Merlis and Schneider, 2011; Sandeep et al., 2014).

Many aspects of the responses of precipitation and circulation strength to warming are not robust across climate model simulations. In particular, there is spread across models in the geographic pattern of the response of tropical precipitation over oceans. Further, while coupled models tend to project a weakening of the WC, the degree of weakening varies greatly across models. These discrepancies motivate evaluation of the mechanisms underlying these features.

The argument for the weakening of the circulation that stems from Equation 1.1 infers the weakening from the changes in specific humidity and precipitation (Held and Soden, 2006). A number of more direct mechanistic explanations have been proposed. In the tropics, the dry static stability increases due to greater warming in the upper atmosphere than in the lower atmosphere, though its effect on the circulation is complicated by changes in latent heating (Ma et al., 2012). The greater warming in the upper troposphere as compared with the lower troposphere is due largely to the differences between moist adiabats at higher and lower temperatures (Miyawaki et al., 2020). Secondarily, there is a direct-CO\(_2\) effect which weakens the circulation due to clouds and water vapor masking the CO\(_2\) radiative forcing (Merlis, 2015).

A mean weakening of the tropical divergent circulation does not guarantee a weak-
ening of the WC because changes in circulation strength need not be uniform (Sandeep et al., 2014). However, a number of mechanisms have been evaluated specifically in the context of the WC. In response to warming, increases in convective heating in the ascent region of the WC are balanced by increases in radiative cooling and dynamical cooling\(^1\) (Knutson and Manabe, 1995). In the descent region, increases in dynamical warming are balanced by changes in radiative cooling. The increase in dynamical cooling is associated with an increase in dry static stability. Increases in dry static stability can be associated with a weaker WC, particularly in the descent region where the role of latent heating is small. Further, changes in dry static stability vary inversely with WC response across models (Sohn et al., 2016). Further, increases in evaporative damping of the surface are larger in the warmer west Pacific than in the cooler east Pacific (Knutson and Manabe, 1995). These differing responses are consistent with a weakening of the zonal sea surface temperature (SST) gradient and WC (Knutson and Manabe, 1995). On the other hand, the ocean thermostat mechanism acts to strengthen the east-west Pacific temperature gradient because of upwelling of cold water in the east Pacific (Clement et al., 1996). Without any other influences, the increased SST gradient due to the cold water upwelling would lead to a wind feedback which strengthened the WC. However, in projections using more realistic simulations, this wind feedback tends act in the opposite direction, consistent with a weaker WC (DiNezio et al., 2009).

The response of the WC strength to warming is an area of substantial model disagreement in both trends and future projections. Further, the response of the WC to warming in observations and reanalysis over recent decades, a strengthening, falls outside of the range of model trends for the same period, a mix of weakening and strengthening (Sohn et al., 2016). Model projections of future warming project a weakening of the WC, though there is substantial spread in the degree of weakening (Vecchi and Soden, 2007).

Precipitation over tropical oceans tends to increase despite a weakening of the

\(^1\)The dynamical cooling term of Knutson and Manabe (1995) is defined using the heat budget and includes temperature advection and adiabatic expansion. It is approximately equal to the stationary vertical advection of potential temperature.
circulation. However, the response of precipitation to warming is not spatially uniform and, further, varies across GCMs (Collins et al., 2013). A number of proposed mechanisms for the precipitation response over tropical oceans have emerged using a variety of frameworks: a moisture budget (Held and Soden, 2006), DSE budget (Muller and O’Gorman, 2011), MSE budget (Chou and Neelin, 2004), and Eq. 1.1 (Chadwick et al., 2013). In general, the response of precipitation to warming is a combination of influences from thermodynamic and dynamic mechanisms.

The proposed thermodynamic contribution states that precipitation response over tropical oceans follows the pattern of control climate precipitation due to increases in water vapor with warming (Held and Soden, 2006). A number of versions of this mechanism exist (the rich-get-richer mechanism, the direct moisture effect (Chou and Neelin, 2004)); we refer to it as the wet-get-wetter mechanism.

The proposed warmer-get-wetter mechanism states that regions with increases in SST relative to the tropical mean tend to have increases in precipitation in response to warming. The warmer-get-wetter mechanism is justified with column-instability arguments. Weak temperature gradients and negligible moisture in the upper troposphere indicate that geographic patterns in column moist static energy tend to follow low-level MSE. Since temperature and humidity are correlated near the surface, SST response is a good proxy for changes in column instability (Xie et al., 2010).

Additionally, precipitation in the control climate is influenced dynamically by surface convergence (SC) (Back and Bretherton, 2009b), and winds comprising this SC are largely proportional to temperature gradients via their corresponding pressure gradients (Lindzen and Nigam, 1987; Back and Bretherton, 2009a).

The large-scale circulation transports energy, and energy budgets have proven useful for diagnosing contributions to the large-scale tropical atmospheric circulation. There are a number of conserved energetic variables in the tropics. Here we make use of the dry static energy (DSE) budget in Chapters 2 and 3 and the moist static energy (MSE) budget in Chapter 4. DSE includes enthalpy and potential energy. The MSE budget combines the DSE budget with a moisture budget. Latent heating due to condensation is a source term in the DSE budget but not the MSE budget.
This term, which includes precipitation, cancels when the column-integrated DSE and moisture budgets combine to form the MSE budget. Consequently, DSE and moisture budgets are useful tools in diagnosing precipitation. The two-mode model of Back and Bretherton (2009b) is based on the DSE budget and includes terms which correspond to the proposed precipitation response mechanisms when applied to climate change.

The precipitation response mechanisms introduced above have emerged from analyses using a variety of frameworks which, in general, assume one mode of vertical motion variability. However, it is clear that two modes are necessary to skillfully capture the geographic variability in vertical velocity (Back and Bretherton, 2009b). Back and Bretherton (2009b) applied the two-mode approximation to the DSE budget which, along with a few other approximations, led to a simple and skillful model of monthly-mean precipitation over tropical oceans.

The namesake approximation in the two-mode model entails splitting vertical velocity, $\omega$, into its two dominant statistical modes of variability. These modes are then combined to create a shallow mode and a deep mode. The shallow mode can be related to SC by mass continuity and the deep mode can be related to SC and SST. The deep mode is related to SST through column instability arguments. The deep mode is related to SC because shallow ascent predisposes the atmosphere to deep ascent due to entrainment, a process studied in Chapter 4. The two-mode model for precipitation represents dynamic mechanisms via changes in vertical velocity and thermodynamic mechanism via changes in dry stratification.

Alternatively, an MSE budget can be used to study the tropical circulation (Neelin and Held, 1987) and its response to warming (Chou and Neelin, 2004). The gross moist stability emerges as a key parameter. The GMS describes the efficiency of a circulation in exporting (or importing) energy. GMS has a number of different definitions, but generally includes advection of a conserved quantity in the numerator and normalizes by circulation strength. GMS tends to be positive in the regions of top-heavy vertical motion, such as the tropical west Pacific, and negative in regions of bottom-heavy vertical motion, such as the equatorial east Pacific. Despite its ubiquity,
GMS is a challenging quantity to observe and to understand. Accurate calculation of vertical GMS requires thermodynamic and vertical velocity profiles across the domain of interest. Further, GMS is sensitive to small perturbations in either of these profiles. In Chapter 3 we apply the two-mode approximation to $\omega$ as it appears in GMS to better understand the spatial pattern of GMS.

Understanding areas of model disagreement is motivation for much climate science research and several aspects of this thesis. Small-scale processes can have a substantial impact on the resulting climate, and differences in small-scale processes may contribute substantially to model spread. In particular, differences in convective parameterizations can have substantial impacts on the resulting climate, especially in the tropics. Convective processes occur on subgrid scales, so have to be parameterized in climate models. One influential process parameterized by convection schemes is entrainment. Entrainment is the process by which convecting air mixes with its environment. Higher entrainment rates tends to suppress deep convection. This deep convection is critical for clouds, precipitation, and the general circulation so the representation of entrainment in convection can have a substantial impact on the simulated climate. Entrainment is very difficult to measure directly, but to the extent that it has been measured, there is evidence that entrainment (and detrainment) processes are very active (Romps, 2010; Yeo and Romps, 2013). Therefore, it is important to understand the sensitivity of a simulated climate to representation of entrainment.

1.2 Chapter 2: The response of tropical precipitation to warming

Chapter 2 builds mechanistic understanding of the response of climatological precipitation to warming over tropical oceans. Precipitation response over oceans is an area of interest because there is model disagreement in GCMs. To further understand the precipitation response, we apply the two-mode model to climate change. When applied to climate change, the two-mode model has terms for the wet-get-wetter
mechanism, the warmer-get-wetter mechanism, and a term representing the role of changes in surface convergence in precipitation response. We are further able to relate the surface convergence term to low-level gradients. Boundary-layer temperature gradients are associated with pressure gradients which can drive a flow from high to low pressure. Surface convergence is the negative of the divergence of the winds, so to the extent that winds are up the temperature gradient, SC is proportional to the Laplacian of boundary-layer temperature. We further are able to relate boundary-layer temperature to SST. Therefore we have three versions of surface convergence: SC, SC as a function of the Laplacian of boundary-layer temperature, and SC as a function the Laplacian of SST. Chapter 2 decomposes the response of precipitation to climate change into contributions from the wet-get-wetter mechanism, the warmer-get-wetter mechanism, and from the various SC approximations in an ensemble of CMIP5 models.

Chapter 2 has been published in the Journal of climate (Duffy et al., 2020). ©March 15, 2020 AMS.

1.3 Chapter 3: Elucidating the gross moist stability

The GMS quantifies the propensity of the atmospheric circulation to respond to a forcing. As a result, GMS is inversely related to precipitation (Raymond et al., 2009). However, it is a difficult quantity to observe and understand. Shifting focus to the current climate, Chapter 3 elucidates the GMS by relating it to SST using observational and reanalysis data. We focus on the vertical component of GMS, though the horizontal component is also important. While there are numerous definitions of GMS, the one we use here is given by

\[ \Gamma_v = \frac{\langle \omega \frac{\partial h}{\partial p} \rangle}{\langle \omega \frac{\partial s}{\partial p} \rangle} \]  

(1.2)
where $\Gamma_v$ denotes the vertical component of GMS, $\langle \cdot \rangle$ denotes a mass-weighted vertical integral, $\omega$ is vertical velocity in pressure coordinates, $s = T + \phi$, and $h = s + q$ is MSE, where $T$ is temperature in energy units, $\Phi$ is geopotential height, and $q$ is specific humidity in energy units. Here we apply the two-mode approximation to $\omega$ as it appears in the GMS expression. Neglecting horizontal variations in $\frac{\partial h}{\partial p}$ and $\frac{\partial s}{\partial p}$, we are able to approximate GMS as a function of SST and SC, SST and Laplacian of boundary-layer temperature, or SST and Laplacian of SST. We evaluate the skill in these approximations in observational and reanalysis data in Chapter 3.

1.4 Chapter 4: The response of the Walker circulation to warming

In order to understand the substantial spread in the response of the Walker circulation to warming in GCMs, Chapter 4 diagnoses the response of the WC to simulated warming using an MSE budget. Further, the influence of entrainment on the WC strength is evaluated in an idealized GCM. WC strength varies inversely with GMS across climates in idealized simulations (Wills et al., 2017). The relationship between changes in WC strength and changes in GMS, horizontal advection, radiation, and surface fluxes is diagnosed in an ensemble of CMIP5 and AMIP models. Further the relationship between GMS and WC strength is studied in idealized GCM simulations of the WC in a control climate and a warm climate with four different entrainment rates.

The convection scheme used in the idealized simulations is an additional contribution of this thesis. The convection schemes is based on the simplified Betts-Miller (SBM) scheme, a quasi-equilibrium convection scheme which relaxes temperature profiles to a moist adiabat in convecting regions. In order to study the effect of entrainment we modify the SBM scheme to create an entraining SBM scheme which relaxes to an entraining adiabat and introduces an entrainment parameter.
Chapter 2

Importance of Laplacian of low-level warming for the response of precipitation to climate change over tropical oceans

Abstract

Several physical mechanisms have been proposed for projected changes in mean precipitation in the tropics under climate warming. In particular, the “wet-get-wetter” mechanism describes an amplification of the pattern of precipitation in a moister atmosphere, and the “warmer-get-wetter” mechanism describes enhanced upward motion and precipitation in regions where the increase in SST exceeds the tropical-mean increase. Studies of the current climate have shown that surface convergence over the tropical oceans is largely driven by horizontal gradients of low-level temperature, but the influence of these gradients on the precipitation response under climate warming has received little attention. Here, a simple model is applied to give a decomposition of changes in precipitation over tropical oceans in 21st-century climate-model projections. The wet-get-wetter mechanism and changes in surface convergence are found to be of widespread importance, whereas the warmer-get-wetter mechanism is primarily limited to negative anomalies in the tropical southern Pacific. Furthermore, surface convergence is linked to gradients of boundary-layer temperature using an atmospheric mixed layer model. Changes in surface convergence are found to be strongly related to changes in the Laplacian of boundary-layer virtual temperature, and, to a lesser extent, the Laplacian of SST. Taken together, these results suggest that a “Laplacian-of-warming” mechanism is of comparable importance to wet get
wetter and warmer get wetter for the response of precipitation to climate change over tropical oceans.

2.1 Introduction

Large changes in tropical precipitation are projected to occur with climate change (Collins et al., 2013), but there are substantial differences between GCMs in the pattern of these changes (Neelin et al., 2006; Kent et al., 2015; Chadwick et al., 2016). Furthermore, the robustness of the response shows little improvement from CMIP3 to CMIP5 (Knutti and Sedlacek, 2013). To better understand the response of precipitation in different models, it is useful to distinguish the contribution to the precipitation response from changes in temperature or humidity (the thermodynamic contribution) and the contribution related to changes in winds or convective mass fluxes (the dynamic contribution). Such a decomposition can be based on the water vapor budget (e.g., Held and Soden, 2006; Seager et al., 2010), an approximate relation involving convective mass flux and low-level specific humidity (e.g., Chadwick et al., 2013), or the dry static energy (DSE) budget (e.g., Muller and O’Gorman, 2011).

The thermodynamic contribution to changes in precipitation results in an amplification of the historical pattern of precipitation (or precipitation minus evaporation) due to increases in water vapor content of the atmosphere with warming. This amplification was referred to as the “direct moisture effect” by Chou and Neelin (2004) and as wet regions getting wetter and dry regions drier by Held and Soden (2006); we will refer to it as the “wet-get-wetter” mechanism for brevity. Wet get wetter becomes evident in the global-scale pattern of precipitation change over ocean in climate-model projections (Held and Soden, 2006; Byrne and O’Gorman, 2015), but it is strongly modified by the dynamic contribution at regional scales over tropical oceans (Chou et al., 2009; Xie et al., 2010; Chadwick et al., 2013).

The dynamic contribution to changes in precipitation is a major source of uncertainties in the projected precipitation response in the tropics (Kent et al., 2015). Chou and Neelin (2004) used the moist static energy (MSE) budget to illustrate how
a dynamic contribution can arise through decreases in convective instability at convective margins (the “upped ante” mechanism) and through changes in gross moist stability in convective regions. Changes in gross moist stability have also been used to argue for a “warmer-get-wetter” mechanism that leads to a positive dynamic contribution to precipitation change in regions where SST increases by more than the tropical mean (Xie et al., 2010; Ma and Xie, 2013; Huang et al., 2013; Huang, 2014). According to this mechanism, a greater increase in SST in a certain region leads to a local decrease in gross moist stability which favors more ascent and precipitation in that region. The influence of the pattern of SST change is found to be surprisingly large compared to wet get wetter over tropical oceans, and this has been argued to be because wet get wetter is largely offset by a weakening of convective mass fluxes (Chadwick et al., 2013).

The mechanisms discussed above for the dynamic contribution to the precipitation response to climate change are based on changes in stability of the atmospheric column. But it is well known for the present climate that SST gradients drive boundary-layer pressure gradients that strongly affect patterns of surface convergence (SC) and precipitation over tropical oceans (Lindzen and Nigam, 1987; Battisti et al., 1999; Schneider and Sobel, 2007; Back and Bretherton, 2009a). In addition, the mechanisms discussed above effectively assume a single mode (e.g., a first baroclinic mode) of vertical motion, but observations and reanalysis show that the shape of vertical motion varies strongly across precipitating regions of the tropical oceans (e.g., Trenberth et al., 2000; Back and Bretherton, 2006; Back et al., 2017). For example, vertical motion in the west Pacific warm pool is typically “top heavy” while vertical motion in the east Pacific ITCZ is typically “bottom heavy” (Back et al., 2017).

Motivated by the need for two modes of vertical motion and the importance of boundary-layer dynamics for precipitation, Back and Bretherton (2009b) (hereafter BB09b) introduced a simple “two-mode” model for precipitation over tropical oceans in the current climate (see also the model of Sobel and Neelin (2006) that shares

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1Changes in SST are linked to changes in vertical velocity with climate change, so warmer get wetter can be considered a dynamic mechanism.
some similar features). The two-mode model uses the DSE budget to relate precipitation to radiative cooling, DSE stratification, and vertical motion. Vertical motion is represented by a combination of deep and shallow modes. The deep mode is tied to SST through an empirical convective instability argument. The shallow mode is tied to SC which is strongly related to gradients of SST, as shown by Lindzen and Nigam (1987) and Back and Bretherton (2009a) (hereafter BB09a). Thus, the two-mode model brings together into one framework many factors that are thought to affect precipitation: radiative cooling (in the DSE budget), mean temperature (which affects the DSE stratification), and column moist stability and SC driven by SST gradients (through the shallow and deep modes).

Here, we adapt and apply the two-mode model of BB09b to better understand the response of precipitation over tropical oceans to climate change in CMIP5 simulations (Taylor et al., 2012). Based on the two-mode model, we are able to quantify the relative contributions of wet get wetter, warmer get wetter, and changes in SC for the precipitation response. Changes in SC are found to be of widespread importance, and so we further investigate them using the atmospheric mixed layer model (MLM) of Stevens et al. (2002) and BB09a. The changes in SC are found to be strongly related to changes in the Laplacian of low-level temperatures, suggesting that a previously-overlooked “Laplacian-of-warming” mechanism is important for projected precipitation changes. A limitation of our approach is that changes in SST (or boundary-layer temperature) are taken as given whereas they are actually part of the coupled atmosphere-ocean response. For example, in a simple model, cloud-shading effects tend to dampen high SST, weakening SST gradients (Peters and Bretherton, 2005; Bretherton and Hartmann, 2009). Further, Naumann et al. (2019) used a simple model to show that shallow circulations can be driven by differential radiative cooling in addition to the influence of the SST gradients emphasized here. Nonetheless it is useful to understand how changes in precipitation are influenced by changes in SST.

We begin by describing the two-mode model of BB09b (Section 2.2), and we

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2Unlike in Sobel and Neelin (2006), the deep mode could not be constrained by the MSE budget because its effective gross moist stability was close to zero.
then apply it to climate projections of precipitation in CMIP5 models to evaluate
the relative contributions of wet get wetter, warmer get wetter, and changes in SC
(Section 2.3). We use the MLM to relate the changes in SC to changes in low-level
temperatures (Section 2.4), and we combine the two-mode model with approximations
for SC to evaluate the role of changes in the $\nabla^2$SST for the precipitation response
(Section 2.5). We further assess our interpretation of the mechanisms for changes
in precipitation using atmosphere-only simulations forced by prescribed SSTs (AMIP
simulations) that isolate the effect of the pattern of changes in SST (Section 2.6)
before giving our conclusions (Section 2.7).

2.2 Two-mode model for tropical precipitation

We use a two-mode model of monthly-mean precipitation that is similar to the model
derived in BB09b but with some improvements. The model’s characteristic approx-
imation is the decomposition of the vertical profile of vertical velocity into its two
leading modes of variability. Differences from the BB09b version of the two-mode
model are described in Appendix A.

The derivation of the two-mode model begins with the time-mean column-integrated
DSE budget. Following BB09b, we neglect horizontal advection of DSE, eddy DSE
fluxes, and surface sensible heat fluxes over tropical oceans so that the DSE budget is
approximated by its dominant terms which relate to latent heating, vertical advection
of DSE, and the radiative flux convergence:

$$LP \simeq \left< \omega \frac{\partial s}{\partial p} \right> - R. \tag{2.1}$$

Here, $P$ is the time-mean surface precipitation rate, $L$ is the latent heat of condensa-
tion neglecting fusion, $\langle \rangle$ is a mass-weighted vertical integral over a tropospheric
column, $\omega$ is the time-mean vertical velocity in pressure coordinates, $s$ is the time-
mean DSE, and $R$ is the time-mean radiative flux convergence. The tropospheric
vertical integral is taken between a nominal tropopause at 100 hPa and surface at
1000 hPa, but due to data availability the radiative flux convergence, $R$, is defined as the net longwave and shortwave fluxes at the surface minus the top-of-atmosphere (TOA), with radiative fluxes defined positive when upwards. In this section, the two-mode model is evaluated using observations and reanalysis, but in later sections it is applied to climate model output. All data are monthly and are first averaged over years to give 12 climatological values at a given location.

### 2.2.1 The two-mode approximation

The vertical velocity as a function of pressure is approximated using its two dominant modes of horizontal and temporal (seasonal) variability as calculated using EOF analysis. The EOF analysis is applied to the climatological monthly $\omega$ between 100 hPa and 1000 hPa using all grid points over the tropical oceans (20°S to 20°N) and including the seasonal cycle. Linear combinations of the two leading EOFs are used to define shallow and deep modes. Without loss of generality, the linear combination for the deep mode is chosen by requiring the deep mode to have zero SC, and the shallow mode is then defined to be orthogonal to the deep mode. This choice of linear combinations allows us to directly relate the shallow-mode amplitude to SC using mass continuity. We approximately impose zero SC in the deep mode by requiring $\omega$ to be equal at the two lowest pressure levels (see Appendix A). The shallow mode has structure $\Omega_s(p)$ and amplitude $o_s(x,y,t)$, and the deep mode has structure $\Omega_d(p)$ and amplitude $o_d(x,y,t)$. The two-mode approximation is written as

$$\omega(x,y,t,p) \simeq o_s(x,y,t) \Omega_s(p) + o_d(x,y,t) \Omega_d(p).$$

(2.2)

The vertical advection of DSE term, $\langle \omega \frac{\partial s}{\partial p} \rangle$, is then written $M_{ss} o_s + M_{sd} o_d$, where $M_{ss} = \langle \Omega_s \frac{\partial s}{\partial p} \rangle$ is the gross dry stratification of the shallow mode, and $M_{sd} = \langle \Omega_d \frac{\partial s}{\partial p} \rangle$ is the gross dry stratification of the deep mode.

We use multiple linear regression to approximate the radiative flux convergence,
as a linear function of the two mode amplitudes,

\[ R(x, y, t) \simeq R_0 + r_s o_s(x, y, t) + r_d o_d(x, y, t), \]  

(2.3)

where \( R_0, r_s, \) and \( r_d \) are constant regression coefficients. We interpret \( R_0 \) to correspond to spatially-averaged radiative heating, while \( r_s \) and \( r_d \) approximately account for the spatially-varying interaction of radiation with clouds and water vapor. Using these radiation regression coefficients, we define gross dry effective stratifications as \( M_{ses} = M_{ss} - r_s \) and \( M_{sed} = M_{sd} - r_d \). Following BB09b, we replace \( M_{ses} \) and \( M_{sed} \) by their horizontal and temporal averages over the tropical oceans such that they are constants (although they do differ between climates and GCMs in the analysis that follows). Using constant \( M_{ses} \) and \( M_{sed} \) is a good approximation because horizontal temperature gradients are weak over tropical oceans. Combining the simplified DSE budget (Eq. 2.1), the two-mode approximation (Eq. 2.2), and the radiative flux convergence approximation (Eq. 2.3) gives

\[ LP(x, y, t) \simeq M_{ses} o_s(x, y, t) + M_{sed} o_d(x, y, t) - R_0. \]  

(2.4)

### 2.2.2 Relating the mode amplitudes to surface quantities

The mode amplitudes \( o_s \) and \( o_d \) are then related to surface quantities, namely relative SST and SC. Relative SST, denoted \( \text{SST}_{\text{rel}} \), is calculated as SST minus SST averaged over the tropical oceans for a given month. SC is calculated using monthly-mean near-surface (10-m) winds. Throughout the paper, horizontal derivatives and the divergence operator are calculated in spherical coordinates using one-sided differences at the coasts and centered finite differences elsewhere.

The continuity equation is used to relate the shallow-mode amplitude \( o_s \) to SC as

\[ o_s(x, y, t) = a_s \text{SC}(x, y, t). \]  

(2.5)

Applying the two-mode approximation for \( \omega \) (Eq. 2.2) to the continuity equation at
the surface, \( SC = \frac{\partial \omega}{\partial p} \), gives that 
\[ SC = \left( \frac{\partial \omega}{\partial p} \right)_{\text{surface}} o_s, \]
where we have used that the deep mode is defined to have zero SC. It follows that the coefficient \( a_s \) is given by 
\[ \left( \frac{\partial \omega}{\partial p} \right)_{\text{surface}} \] 
which we evaluate using \( \Omega_s \) at the two pressure levels that are nearest the surface.

The deep-mode amplitude \( o_d \) is approximated by a multiple linear regression with \( \text{SST}_{\text{rel}} \) and \( SC \),
\[ o_d \approx b_{\text{SST}} \text{SST}_{\text{rel}} + b_{SC} \text{SC} + b_0, \]
where \( b_{\text{SST}}, b_{SC}, \) and \( b_0 \) are regression coefficients. The regression coefficients are calculated using locations and months for which climatological \( \text{SST}_{\text{rel}} \) is positive because we want to broadly focus on precipitating regions but the resulting estimate of \( o_d \) is relatively insensitive to this choice. By including \( SC \) in the deep-mode regression we are effectively including the shallow-mode amplitude to account for the favorable effect of shallow ascent and moistening of the environment for deep convection. As discussed in detail in Appendix A, BB09b used a different deep-mode approximation, but we find that the approximation given by Eq. 2.6 is more consistent with the relationship inferred from reanalysis and observations.

2.2.3 Expression for precipitation and comparison to observations

An expression for precipitation comes from combining Eq. 2.4 with Eqs. 2.5, 2.6, but this can give negative precipitation because of the approximations used (e.g., Eq. 2.6 is not accurate for negative \( \text{SST}_{\text{rel}} \)). Therefore, we include a Heaviside function, \( H() \), to prevent the approximations from giving negative precipitation. The final model for precipitation over tropical oceans, referred to the two-mode model, is given by

\[ LP \approx H(\chi) \chi, \]
\[ \chi(x, y, t) = M_{\text{sens}} a_s SC(x, y, t) + M_{\text{sed}} [b_{\text{SST}} \text{SST}_{\text{rel}}(x, y, t) + b_{SC} SC(x, y, t) + b_0] - R_0. \]

(2.7)
Notice that SC and SST_{rel} are the only spatially varying inputs. The parameters $M_{ses}$, $a_s$, $M_{sed}$, $b_{SST}$, $b_{SC}$, $b_0$, and $R_0$ are constants in our evaluation based on reanalysis data, and they only vary between climates and GCMs in our climate-model analyses. The coefficient $a_s$ relates the shallow-mode amplitude to SC; $M_{ses}$ and $M_{sed}$ combine the dry stratification of the atmosphere with the shallow- and deep-mode structures, respectively; and $b_{SST}$, $b_{SC}$, and $b_0$ are empirical linear regression coefficients relating the deep-mode amplitude to SST_{rel} and SC.

We evaluate the two-mode model over August 1999 through July 2009 using monthly ERA-Interim reanalysis data (Dee et al., 2011) for vertical velocity, temperature, geopotential, shortwave and longwave radiative fluxes at the surface and top of atmosphere; monthly QuikSCAT observational data for winds used to calculate SC (NASA, 2012a,b); and monthly NOAA optimal interpolation SSTs (Reynolds et al., 2002). The time period was chosen based on availability of QuikSCAT data. Here and throughout the paper, all data are linearly interpolated to a $1^\circ \times 1^\circ$ horizontal grid and an evenly spaced pressure grid. Gridboxes with nonzero land are masked based on the land fraction variable from the GFDL-CM3 GCM, and the same land mask is used throughout. The deep mode has ascent throughout the troposphere with the strongest ascent in the upper troposphere, while the shallow mode has ascent only in the lower troposphere and weak descent in the upper troposphere (Fig. A1a).

We first compare spatially-smoothed time-mean precipitation from the two-mode model to GPCP (Adler et al., 2003) over August 1999 through July 2009 (Fig. 2-1). Throughout the paper, where spatial smoothing is indicated it is done by convolving the data with a two-dimensional, nine-point averaging filter. Also throughout the paper, precipitation from the two-mode model is evaluated using as inputs climatological monthly-mean SC and SST_{rel} averaged over all years for each month of the year. As compared to monthly-mean GPCP (Fig. 2-1b), the two-mode model accurately captures the distribution of monthly-mean precipitation (Fig. 2-1b) with a RMSE of 2.08 mm day$^{-1}$. The original two-mode model from BB09b (Fig. 2-1c) gives a similar distribution of precipitation, but it has a higher RMSE of 2.30 mm day$^{-1}$. The seasonal cycle is included in the RMSE values that we report, but the RMSE of
Figure 2-1: Mean precipitation over August 1999 through July 2009 from (a) GPCP observations, (b) two-mode model used here (smoothed), and (c) two-mode model of BB09b (smoothed). Contour interval: 1 mm day$^{-1}$. Values of RMSE in (b) and (c) are of the climatological monthly precipitation relative to GPCP precipitation shown in (a), before smoothing.

The annual mean is actually higher for the new version of the model than the BB09b version of the model. The most important difference between the new and BB09b versions of the two-mode model is the deep-mode amplitude approximation. When the deep-mode amplitude is plotted as a function of SST$_{rel}$ and SC, it is clear that the new version of the two-mode model is preferable to the BB09b version (Fig. A2), so the new version is used subsequently.

### 2.3 Precipitation response to climate change

We apply the two-mode model to simulations of different climates from an ensemble of 20 coupled GCMs from CMIP5 [Taylor et al. 2012] listed in Table S1. The response to climate change, denoted by $\Delta$, is defined as the difference between a historical and a warmer climate. The historical climate is the time mean of the historical simulation over 1980-1999, and the warmer climate is the time mean of the RCP8.5 simulation over 2080-2099, with some exceptions. For each GCM and for each of the historical
and warmer climates, the vertical modes and parameters of the two-mode model are calculated using the GCM data for that climate following the approach described in the previous section. As shown in Fig. A1b, the vertical modes in the ensemble mean are similar to those obtained from reanalysis, and they shift upwards with climate warming consistent with a general upward shift of the general circulation (Singh and O’Gorman 2012) and with a deepening of the maximum level of convection (Chou et al. 2013). Monthly precipitation from the two-mode model is then calculated for each GCM and climate using climatological monthly SST_{rel} and SC from the GCMs as inputs. To give greater emphasis on the aspects that are common amongst models, we take the ensemble mean across GCMs prior to calculating RMS, RMSE and \( r^2 \) values.

The two-mode model applied to the historical climate in the GCMs accurately reproduces the time- and ensemble-mean precipitation as simulated by the GCMs (RMSE = 1.31 mm day\(^{-1}\)). The two-mode model also accurately reproduces the
changes in time-mean precipitation ($\Delta P$) with climate warming both in the ensemble mean, as shown in Fig. 2-2, and in individual GCMs, as shown for the MPI-ESM-MR model in Fig. 2-3. We chose MPI-ESM-MR to show in Fig. 2-3 because it is an example of a model with a $\Delta P$ that is quite different from the ensemble mean. For MPI-ESM-MR, the strong precipitation increase in the western Pacific extends farther south, there is little change in the ITCZ region of the central Pacific, and the changes in the Indian and Atlantic basins are larger as compared to the ensemble mean. These differences between $\Delta P$ for MPI-ESM-MR and the ensemble mean are broadly captured by the two-mode model.

To evaluate the contributions of wet get wetter, warmer get wetter, and changes in SC, we recalculate $\Delta P$ from the two-mode model allowing only the relevant terms in the expression for $\chi$ (Eq. 2.7) to respond to climate change. In the framework of the DSE budget, the increase in DSE stratification, $-\frac{\partial s}{\partial p}$, with warming corresponds to a wet-get-wetter mechanism (Muller and O’Gorman [2011]), and this increase dominates the changes in $M_{ses}$ and $M_{sed}$. Therefore, to evaluate the wet-get-wetter contribution, only $M_{ses}$ and $M_{sed}$ are allowed to respond to climate change in Eq. 2.7, while the mean of historical and RCP8.5 values are used for the other parameters and for SC and SST$_{rel}$. To evaluate the warmer-get-wetter contribution, only SST$_{rel}$ is allowed to
respond. To evaluate the contribution of \( \Delta SC \), only SC, which appears in both the shallow- and deep-mode amplitudes, is allowed to respond. The sum of these three contributions to \( \Delta P \) is not identically equal to the \( \Delta P \) given by the two-mode model because of small changes in the parameters \((a_s, b_{SST}, b_{SC}, b_0, \text{ and } R_0)\) and in the Heaviside function, but the sum of the three contributions is a good approximation to the total two-mode model response (RMSE of 0.20 mm day\(^{-1}\)).

The wet-get-wetter mechanism tends to increase the magnitude of precipitation where it is large in the historical climate (Figs. 2-2d, 2-3d). The warmer-get-wetter mechanism is primarily limited to part of the south Pacific where it contributes a strong negative precipitation change (Figs. 2-2e, 2-3e). Notably, much of the structure of \( \Delta P \) is due to changes in SC, particularly in the Pacific in the ensemble mean (Fig. 2-2f) and in all basins for MPI-ESM-MR (Fig. 2-3f). A negative contribution from changes in SC partly offsets the positive wet-get-wetter contribution in some regions, but there are also regions where the SC contribution is positive. Overall, we find that the importance of the three contributions is relatively similar across models, but that they can combine to give different patterns of \( \Delta P \). In the ensemble mean, the RMS values of the contributions are 0.53 mm day\(^{-1}\) for wet get wetter, 0.40 mm day\(^{-1}\) for warmer get wetter, and 0.63 mm day\(^{-1}\) for SC\(^{-1}\).

The warmer-get-wetter and wet-get-wetter mechanisms have been discussed extensively in the literature. It is therefore notable that we find a strong contribution of changes in SC and a relatively limited contribution of warmer get wetter in our decomposition.

### 2.4 Relationship between changes in SC and the Laplacian of low-level temperatures

The importance of changes in SC in setting the precipitation response motivates us to better understand what determines the pattern and magnitude of the changes in SC. The ensemble mean of the change in SC features a prominent increase in the equatorial
Pacific flanked by bands of decreases farther south and north (Fig. 2-4a). Previous work suggests that low-level winds in the present climate are strongly affected by SST gradients. The SST gradients imprint on the boundary-layer temperature gradients which, by hydrostatic balance, induce horizontal pressure gradients that help to drive low-level winds (Lindzen and Nigam, 1987; Battisti et al., 1999). In their simplest form, these arguments suggest that SC should be related to the curvature or Laplacian of SST (e.g., Schneider and Sobel, 2007). For climate change simulated by the CMIP5 models, we find that the pattern of change in SC is related to the spatially-smoothed pattern of change in $\nabla^2$SST ($\Delta \nabla^2$SST) with $r^2 = 0.39$ for the ensemble mean (Fig. 2-4a and b). By contrast, $\Delta$SST$_{rel}$ is more strongly weighted to the Southern Hemisphere and is more weakly correlated with $\Delta$SC ($r^2 = 0.26$) and $\Delta \nabla^2$SST ($r^2 = 0.21$). The relatively high correlation of changes in SC with changes in $\nabla^2$SST suggests a possible role for convergence driven by boundary-layer pressure gradients related to gradients of SST. We next use the MLM developed in Stevens et al. (2002) and BB09a to further investigate the relationship between changes in SC and changes in low-level temperatures.
The MLM is based on a horizontal momentum balance involving pressure gradients, Coriolis acceleration, surface drag, and downward momentum mixing from the free troposphere. According to the MLM, the bulk boundary-layer zonal wind $U$ and meridional wind $V$ are given by

$$
U \approx \frac{U_{850} \epsilon_i \epsilon_e + V_{850} f \epsilon_e - \rho_0^{-1} \left( f \frac{\partial p_s}{\partial y} + \epsilon_i \frac{\partial p_s}{\partial x} \right)}{\epsilon_i^2 + f^2},
$$

$$
V \approx \frac{V_{850} \epsilon_i \epsilon_e - U_{850} f \epsilon_e + \rho_0^{-1} \left( f \frac{\partial p_s}{\partial x} - \epsilon_i \frac{\partial p_s}{\partial y} \right)}{\epsilon_i^2 + f^2},
$$

(2.8)

where $U_{850}$ and $V_{850}$ are winds at 850 hPa, $\rho_0$ is a reference density set to 1.15 kg m$^{-3}$, $f$ is the Coriolis parameter, $p_s$ is surface pressure, $\epsilon_e$ is a tuned parameter related to entrainment mixing at the top of the boundary layer, and $\epsilon_i$ is a tuned parameter related to both this entrainment mixing and surface friction. The fields $U_{850}$, $V_{850}$, and $p_s$ are taken from GCM output, with $p_s$ gradients spatially smoothed to reduce noise. For simplicity, we identify the convergence of the mixed layer winds in Eq. 2.8 with the convergence at the surface and refer to it as SC(MLM). Using the values for $\epsilon_e$ and $\epsilon_i$ from BB09a results in SC that is too strong as compared to GCM SC for the historical climate. Doubling the value of the parameters $\epsilon_e$ and $\epsilon_i$ used by BB09a gives roughly the right magnitude of SC in the historical climate compared to the GCMs, so we set $\epsilon_e = 4 \times 10^{-5}$ s$^{-1}$ and $\epsilon_i = 7 \times 10^{-5}$ s$^{-1}$.

We compare SC(MLM) to the ensemble mean of SC from eleven of the CMIP5 GCMs, which we refer to as SC(GCM). The GCMs used are listed in the middle column of Table S1. In the historical climate, SC(MLM) is in reasonable agreement with SC(GCM) in pattern and magnitude (not shown). The pattern of $\Delta$SC(MLM) generally agrees with $\Delta$SC(GCM), but the magnitude of the dominant negative-positive-negative feature in the Pacific is generally weaker and narrower in $\Delta$SC(MLM) than in $\Delta$SC(GCM) (Fig. 2-5a,b).

We next derive two approximations for SC from the MLM: one that involves the

---

4CMCC-CESM, CMCC-CM, CMCC-CMS, CSIRO Mk3.6.0, and MPI-ESM-MR are excluded because surface specific humidity was not available for these models, and CanESM2, CNRM-CM5, MIROC5, and MIROC-ESM are excluded because of pronounced spectral ringing in the surface pressure field.
Laplacian of boundary-layer virtual temperature $[SC(\nabla^2 T_v)]$ and one that involves the Laplacian of SST $[SC(\nabla^2 \text{SST})]$.

### 2.4.1 Approximation 1: SC $\nabla^2 T_v$

The first approximation relates SC to horizontal temperature variations in the boundary layer. Following BB09a, the boundary layer is assumed to include the surface to the top of the trade inversion. The nominal boundary-layer top is located at the mean height of the 850 hPa pressure surface, $\bar{z}_{850}$, and the local pressure at $z = \bar{z}_{850}$ is denoted $p_i$. Combining the hydrostatic relation and the ideal gas law gives that

$$p_s = p_i e^{\frac{g}{R_d} \int_{BL} T_v^{-1} dz},$$  

where $R_d$ is the gas constant for dry air, $g$ is the acceleration due to gravity, $\int_{BL} dz = \int_0^{\bar{z}_{850}} dz$ is an integral over the boundary layer, and $T_v$ is virtual temperature evaluated as $T_v = (1 + 0.61q) T$ where $q$ is specific humidity.

BB09a showed that the contributions to SC from horizontal gradients of $p_i$ and downward mixing of the winds at 850 hPa partly offset due to geostrophic balance at $\bar{z}_{850}$. The sum of these contributions to SC was found to be much smaller than the contribution from horizontal gradients in the pressure difference across the boundary layer. Therefore, we seek an approximation that neglects horizontal variations in $p_i$ and terms involving the winds at 850 hPa. Taking the horizontal gradient of Eq. 2.9, neglecting horizontal variations of $p_i$, and again making use of Eq. 2.9 gives that

$$\frac{\partial p_s}{\partial x} \approx -\frac{p_s g}{R_d} \int_{BL} \frac{\partial T_v}{\partial x} \frac{dz}{T_v^2},$$

$$\frac{\partial p_s}{\partial y} \approx -\frac{p_s g}{R_d} \int_{BL} \frac{\partial T_v}{\partial y} \frac{dz}{T_v^2}. \tag{2.10}$$

Substituting Eq. 2.10 in Eq. 2.8 and neglecting terms involving $U_{850}$ and $V_{850}$ gives an expression for SC in terms of horizontal gradients of boundary-layer virtual tem-
perature:

\[ SC(T_v) = -\nabla \cdot \mathbf{V}(T_v), \]

where

\[ U(T_v) \approx \frac{p_s g}{R_d \rho_0 (\epsilon_i^2 + f^2)} \left( f \int_{BL} \frac{\partial T_v}{\partial y} \frac{dz}{T_v^2} + \epsilon_i \int_{BL} \frac{\partial T_v}{\partial x} \frac{dz}{T_v^2} \right) \]

\[ V(T_v) \approx -\frac{p_s g}{R_d \rho_0 (\epsilon_i^2 + f^2)} \left( f \int_{BL} \frac{\partial T_v}{\partial x} \frac{dz}{T_v^2} - \epsilon_i \int_{BL} \frac{\partial T_v}{\partial y} \frac{dz}{T_v^2} \right), \]

(2.11)

Explicit evaluation of the convergence of the winds in Eq. 2.11 shows that \( SC(T_v) \) includes a term involving the Laplacian of \( T_v \) and that some of the other terms cancel each other. The Laplacian term is dominant, and the first approximation is simply that term, given by

\[ SC(\nabla^2 T_v) \approx -\frac{p_s g \epsilon_i}{R_d \rho_0 (\epsilon_i^2 + f^2)} \int_{BL} \frac{\nabla^2 T_v}{T_v^2} dz, \]

(2.12)

where \( \nabla^2 T_v \) is the Laplacian of virtual temperature. Lindzen and Nigam (1987) also found that the Laplacian term was important for SC, but they found that a beta convergence term related to variations in \( f \) with latitude was of similar importance. The beta convergence term makes a much smaller contribution in our MLM formulation because we use a stronger frictional coefficient that is more similar to the ones used by Stevens et al. (2002) and Back and Bretherton (2009a).

In practice, we evaluate \( \int_{BL} \) in pressure coordinates by using the hydrostatic relation and approximating the upper limit as 850 hPa rather than \( p_i \). \( SC(\nabla^2 T_v) \) accurately reproduces \( SC(\text{MLM}) \) in pattern and magnitude in the historical climate (not shown) and in the response to climate change (Fig. 5b,c).

### 2.4.2 Approximation 2: \( SC(\nabla^2 \text{SST}) \)

We further approximate \( SC(\nabla^2 T_v) \) (Eq. 2.12) to get an expression for SC proportional to \( \nabla^2 \text{SST} \). As noted by Lindzen and Nigam (1987), the horizontal pattern of temperature imprinted by the SST decays with height throughout the boundary layer. For simplicity, we assume that the temperature pattern decays linearly with
height through the boundary layer as $\nabla^2 T_v \simeq \nabla^2 SST \left(1 - \frac{z}{z_{850}}\right)$. We approximate $T_v^2 \simeq SST^2$ in the denominator of Eq. 2.12, which is accurate to the extent that the temperature difference across the boundary layer is much smaller than the SST. Plugging these approximations into Eq. 2.12 gives

$$\text{SC}(\nabla^2 \text{SST}) \simeq -\frac{3\hat{z}_{850} p_s g \epsilon_i}{4 R_d \rho_0 (\epsilon_i^2 + f^2) SST^2} \nabla^2 \text{SST}. \quad (2.13)$$

As compared to SC($\nabla^2 T_v$), SC($\nabla^2 \text{SST}$) reproduces much of the spatial pattern but is too strong in magnitude for both the historical climate (not shown) and the response to climate change (Fig. 2-5c,d). As compared to SC(GCM), SC($\nabla^2 \text{SST}$) captures several of the main features of the response to climate change but also has substantial errors (Fig. 2-5a,d).

Overall, the results in this section show that the projected effect of climate change on SC over tropical oceans is largely driven by changes in the Laplacian of boundary-layer temperatures, and the main features of the SC response are related to changes in the $\nabla^2 \text{SST}$. Given the widespread importance of the change in SC for the precipitation response, this suggests an important “Laplacian-of-warming” mechanism that acts alongside wet get wetter and warmer get wetter for the precipitation response over tropical oceans.

### 2.5 Estimating precipitation response using approximations for SC

We next examine the extent to which the two-mode model captures changes in precipitation when SC is approximated rather than taken from the GCMs. In particular, replacing SC(GCM) in the two-mode model with SC($\nabla^2 T_v$) or SC($\nabla^2 \text{SST}$) gives a model for precipitation whose only spatially varying inputs are SST and boundary-layer virtual temperatures (when SC($\nabla^2 T_v$) is used) or just SST (when SC($\nabla^2 \text{SST}$) is used). Only the GCMs for which the MLM can be calculated are included in the analysis (Table S1), but this subset of GCMs gives similar results for the simulated
Figure 2-5: Ensemble-mean response of SC to climate change from (a) GCMs, (b) MLM given by Eq. 2.8, (c) SC($\nabla^2 T_v$) given by Eq. 2.12, and (d) SC($\nabla^2 SST$) given by Eq. 2.13. Contour interval: $5 \times 10^{-7}$ s$^{-1}$. Zero contour denoted by thick black contour. The MLM and approximations are applied to each GCM and climate separately. The subset of models used for this figure is given in the middle column of Table S1.

$\Delta P$ and the two-mode model prediction for $\Delta P$ with SC(GCM) (Fig. 2-6a,b) as compared to the full set of GCMs (Fig. 2-2a,b). The regression coefficients in the multiple regression for the deep-mode amplitude ($b_0$, $b_{SST}$, and $b_{SC}$) are recalculated for each of the approximations of SC.

When $\Delta SC(MLM)$ is used instead of $\Delta SC(GCM)$ to estimate $\Delta P$, the positive anomaly in the east Pacific weakens, which contributes to an increase in RMSE of 0.29 mm day$^{-1}$ (Fig. 2-6b,c). The approximations made to SC(MLM) to give SC($\nabla^2 T_v$) increase the RMSE of $\Delta P$ by only 0.05 mm day$^{-1}$ and the further approximations made to SC($\nabla^2 T_v$) to give SC($\nabla^2 SST$) increase the RMSE of $\Delta P$ by 0.06 mm day$^{-1}$ (Fig. 2-6d,e). The main features of the ensemble-mean $\Delta P$ are qualitatively captured by all of the SC approximations: a positive anomaly in the northern part of the Indian ocean; an elongated-c-shaped, positive anomaly in the Pacific ocean; a negative anomaly in the south Pacific; and a small, positive anomaly in the Atlantic ocean.
(Fig. 2-6). This is particularly notable for $\Delta P$ using the SC($\nabla^2$SST) approximation (Fig. 2-6e) since SST is the only spatially varying input.

However, errors in $\Delta P[\text{SC}(\nabla^2\text{SST})]$ at each gridpoint are substantial enough that a more statistical model would not necessarily find that SC($\nabla^2$SST) strongly affects $\Delta P$. To illustrate this, consider the shallow mode, whose amplitude depends only on SC (or one of its approximations) through the coefficient $a_s$ in the two-mode model (Eq. 2.5). We generally calculate $a_s$ from mass continuity, so it does not change as SC is approximated. However, in the version of the two-mode model with SC($\nabla^2$SST), if $a_s$ is calculated from a linear regression so that $a_s$ can change as SC is approximated, then the role of SC becomes muted and the precipitation response, including a faint elongated-c shape, is largely dominated by the wet-get-wetter and warmer-get-wetter mechanisms (compare Figs. 2-6d, 2-6f). The fact that SC becomes muted when SC is approximated as SC($\nabla^2$SST) and $a_s$ is chosen by regression emphasizes that errors in SC($\nabla^2$SST) are important when using it to calculate the response of precipitation to climate change. The importance of errors in SC($\nabla^2$SST) can also be inferred from the decrease in the deep mode regression coefficient $b_{SC}$ as SC is approximated. For example, in historical CMIP5 simulations, using SC($\nabla^2T_v$) instead of SC decreases $b_{SC}$ by nearly a factor of two, and using SC($\nabla^2$SST) instead of SC($\nabla^2T_v$) reduces $b_{SC}$ by a factor of seven (Table S2).

2.6 AMIP analysis

The analyses in the previous sections suggest that changes in SC related to changes in the Laplacian of low-level warming are important for the response of precipitation to climate change. However, the simulations used involve changes in radiative absorbers and a sizable increase in mean temperature which can also affect the circulation and precipitation. Here, we further test our interpretation by using AMIP simulations.

When SC($\nabla^2$SST) is used, the structure of the wet-get-wetter mechanism differs from that shown in Fig. 2d because the shallow- and deep-mode amplitudes are further approximated, and that the subset of models used here gives a slightly different warmer-get-wetter pattern than that shown in Fig. 2e.
Figure 2-6: Ensemble-mean response of precipitation to climate change from (a) GCMs, (b) two-mode model, (c) two-mode model with SC(MLM) as given by Eq. 2.8, (d) two-mode model with SC($\nabla^2 T_v$) as given by Eq. 2.12, (e) two-mode model with SC($\nabla^2$SST) as given by Eq. 2.13, and (f) from the two-mode model with SC($\nabla^2$SST) and with the shallow-mode coefficient from a regression. The subset of models used for this figure is given in the middle column of Table S1. Contour interval: 0.5 mm day$^{-1}$. Zero contour denoted by thick black contour.

to isolate the effects due to changes in the pattern of surface warming with climate change.

The AMIP simulations include a control simulation (AMIP\_control), a simulation with a spatially uniform SST increase of 4K (AMIP\_4K), and a simulation with a spatially patterned SST increase (AMIP\_future), of the years 1979 through 2008. The response to a “pattern-only” change in SST (i.e. with no mean increase in SST) will be referred to as AMIP\_pattern and is calculated as the normalized AMIP\_future response minus the normalized AMIP\_4K response. The response in each simulation is normalized by the change in tropical-mean SST, which differs slightly between simulations. This approach effectively assumes that the AGCM’s responses are linear with surface warming. Seven AGCMs for which the necessary simulations and variables were available were used for the analysis (Table S1).

The two-mode model approximately reproduces $\Delta P$ for AMIP\_pattern in the ensemble mean (Figs. 2-7a,b). Wet get wetter is negligible (Fig. 2-7d) because there is no mean warming and thus there is little change in the gross effective dry stratifica-
tions $M_{acs}$ and $M_{sed}$ whose changes we use to represent the wet-get-wetter mechanism. Absent wet get wetter, $\Delta P$ is dominated by the contributions from warmer get wetter (Fig. 2-7e) and $\Delta SC$ (Fig. 2-7f), and the overall structure of $\Delta P$ is similar to that of the $\Delta SC$ contribution. Further, we evaluate the contribution to the precipitation response from $\Delta SC$ when it is replaced with each of the SC approximations: $\Delta SC(\nabla^2 T_v)$ and $\Delta SC(\nabla^2 SST)$. As was done previously for Fig. 2-6, the regression coefficients in the multiple regression for the deep-mode amplitude ($b_0$, $b_{SST}$, and $b_{SC}$) are recalculated for each of the approximations of SC. The contribution to $\Delta P$ from $\Delta SC(\nabla^2 T_v)$ has a weaker magnitude than the contribution from $\Delta SC(GCM)$, but the pattern is similar (Fig. 2-7g). The contribution to $\Delta P$ from $\Delta SC(\nabla^2 SST)$ has a similar pattern in the Pacific although there are differences over the Indian and Atlantic oceans (Fig. 2-7h).

The two-mode model also approximately captures the precipitation response for both AMIP_4K and AMIP_future (Figs. S1 and S2). The AMIP_4K response (Fig. S1) has a substantial wet-get-wetter contribution with no warmer-get-wetter contribution (because $\Delta SST_{rel}$ is zero) and relatively weak changes in SC (consistent with zero imposed changes in the $\nabla^2 SST$). Differences between AMIP_4K $\Delta P$ and the wet-get-wetter contribution do not resemble a weakening of the climatological precipitation pattern, and thus a weakening of the tropical divergent circulation due to spatial mean warming does not seem to be important for precipitation changes in our framework (see Supplemental Text in Appendix B for further discussion). The decomposition of the AMIP_future precipitation response (Fig. S2) is similar to what was discussed for the coupled models under RCP8.5.

Overall, the AMIP simulations support our interpretation of the contributions to changes in precipitation over tropical oceans: mean warming affects the precipitation response primarily via wet get wetter, while the pattern of changes in SST acts via warmer get wetter and a new “Laplacian-of-warming” mechanism.
Figure 2-7: Ensemble-mean response of precipitation to a pattern-only change in SST (AMIP_pattern): (a) AGCMs, (b) two-mode model, (c) sum of warmer-get-wetter, wet-get-wetter, and SC(GCM) contributions, (d) wet-get-wetter contribution, (e) warmer-get-wetter contribution, (f) SC contribution based on SC(GCM), (g) SC contribution based on SC($\nabla^2 T_v$), and (h) SC contribution based on SC($\nabla^2$SST). The subset of models used is given in the right column of Table S1. The response is normalized by the change in tropical-mean SST. Contour interval: 0.125 mm day$^{-1}$ K$^{-1}$. Zero contour denoted by thick black contour.

2.7 Conclusions

We have analyzed precipitation projections from CMIP5 over the tropical oceans using a simple model for precipitation based on two modes of vertical motion. The need for two modes to describe vertical motion in the tropical atmosphere is well known based on analyses of the current climate, and here two modes are also used to analyze climate change. The two-mode model leads to a physical decomposition of the response of precipitation to climate change in which wet get wetter, warmer get wetter, and changes in SC are all needed to give the change in precipitation. Changes in SC and wet get wetter are of widespread importance, whereas warmer get wetter is primarily limited to the southern tropical Pacific. We went on to show using momentum balance in the boundary layer that the changes in SC can be approximated...
as proportional to changes in $\nabla^2 T_v$ and, to a lesser extent, $\nabla^2 \text{SST}$. AMIP simulations were used to isolate the effect of the pattern of SST change versus the effect of the mean warming. The AMIP simulations were found to support our interpretation of an important role for changes in SC driven by changes in the Laplacian of low-level temperature.

Our results reveal that a “Laplacian-of-warming” mechanism is of widespread importance in precipitation projections in addition to the warmer-get-wetter and wet-get-wetter mechanisms. The Laplacian-of-warming mechanism may not have been distinguished from warmer get wetter previously because $\text{SST}_{\text{rel}}$ and $\nabla^2 \text{SST}$ necessarily have some similarities in terms of spatial pattern and both can help to place precipitation maxima on SST maxima (cf. Schneider and Sobel, 2007). For example, Xie et al. (2010) found that the pattern of SST change was important for the response of precipitation, consistent with our results, but they attributed this to the dominant effect of changes in $\text{SST}_{\text{rel}}$ on changes in gross moist stability. Our results show that changes in $\text{SST}_{\text{rel}}$ and changes in $\nabla^2 \text{SST}$ nonetheless have quite different structure, particularly in terms of differences between the Northern and Southern Hemispheres (Fig. 2-4b,c), and we find correspondingly distinct contributions from warmer get wetter and Laplacian of warming. Ultimately, the fundamental difference between warmer get wetter and Laplacian of warming is between influences on precipitation that depend on SST changes relative to the tropical-average warming (through $\text{SST}_{\text{rel}}$) and relative to the average warming in the immediate vicinity (through $\nabla^2 \text{SST}$).

The importance of the Laplacian-of-warming mechanism poses a challenge for climate modeling. Our findings indicate that the responses of both SST and Laplacian of SST are important in the setting precipitation response, implying that climate models must accurately simulate the details of SST response in order to accurately simulate the precipitation response.

One limitation of our results is that the errors in estimating changes in SC from $\nabla^2 \text{SST}$ are substantial. Indeed the role of SC($\nabla^2 \text{SST}$) becomes muted if the constraint of mass continuity is neglected and all coefficients are chosen by regression (see Fig. 6f). In general, the accuracy is higher if boundary-layer $\nabla^2 T_v$ is used.
instead of $\nabla^2 \text{SST}$.

In addition to the climate-model projections studied here, it would be interesting to apply the two-mode model to historical trends in precipitation and SSTs in climate-model simulations and observations. We have not focused in particular on the ITCZ, but recent work suggests that $\nabla^2 \text{SST}$ may be important in setting the width of the ITCZ consistent with our results (Byrne and Thomas, 2019). Atmospheric energy balance, dictated in part by the ocean circulation and remote forcing, is thought to be important to ITCZ position (Kang et al., 2009; Schneider et al., 2014; Green and Marshall, 2017), and using the two-mode model to address this would require coupling it to, for example, a mixed layer ocean rather than taking SST as given. Finally, we have focused exclusively on ocean regions, but a model of comparable simplicity to the two-mode model would also be useful for analysis of precipitation changes over tropical land.
Chapter 3

Relating the vertical Gross Moist Stability to SST and SST gradients

Abstract

The gross moist stability (GMS) emerges as a key parameter in theories of the tropical circulation. The GMS summarizes the relationship between atmospheric net energy input and the large-scale circulation by quantifying the energy import or export of a circulation. Despite its importance, GMS is a difficult quantity to understand and to observe. Here we focus on understanding the vertical component of monthly GMS using observations. Many theories about the vertical GMS make assumptions about the shape of the vertical motion profile. However, it is known from reanalysis, numerical models, and observations that vertical motion profile shapes can vary dramatically in time and on large spatial scales. Therefore we use a definition of vertical GMS which accounts for spatial variations in vertical motion profiles. We find that geographic patterns of GMS are dominated by geographic patterns of vertical motion. Further, we examine the extent to which vertical motion can be related to sea surface temperature (SST) and SST gradients over the tropical oceans using a simple, two-mode model of vertical motion. This simple model is based partially on column instability ideas and partially on the idea that SST gradients drive surface convergence (SC) by imprinting hydrostatically on the boundary layer. Consequently, the simple model estimates the shape of the vertical motion profile as a function of the SST and SST gradients. Since spatial variations in vertical GMS are dominated by spatial variations in vertical motion, we are able to approximate the vertical GMS using SST and SST gradients. We find that these estimates of GMS work well in the mean and seasonal cycle, especially given their simplicity.
3.1 Introduction

Energy conservation principles can be useful in studying the tropical atmosphere. The gross moist stability (GMS) emerges as a key parameter in many theories of the tropical atmosphere that are based on moist energy conservation. GMS describes the efficiency of a circulation’s response to net energy input into the atmosphere. GMS was first introduced by Neelin and Held (1987) as a key parameter in their model of tropical convergence based on moist static energy (MSE) conservation. However, at the time, GMS was aptly described as "a convenient way of summarizing our ignorance of the details of the convective and large scale transients" when viewing the atmosphere from an energetic perspective (Neelin and Held, 1987). Their description remains accurate more than 30 years later. There are several factors that contribute to the elusiveness of GMS. Firstly, it is a difficult quantity to define, with many different definitions. Further, it is a difficult quantity to calculate and observe. GMS requires accurate thermodynamic and vertical velocity profiles, and is sensitive to small errors. We address the difficulty to calculate and observe GMS by deriving and evaluating an expression for vertical GMS that depends solely on the sea surface temperature (SST).

Precipitation minus evaporation can be written as proportional to the entropy forcing divided by the GMS, so the GMS modulates the relationship between precipitation and net energy input (Raymond et al., 2009). In other words, the GMS represents the efficiency by which overturning circulations export or import energy. GMS has been defined in a variety of ways but, in general, relates a conserved thermodynamic variable to the strength of moist convection. It is typically written as a ratio with advection of a thermodynamic quantity in the numerator and a normalizing quantity in the denominator, though the details vary. The GMS can be decomposed into its vertical component and horizontal components because the advection in the numerator can be separated in vertical and horizontal contributions. Here we study the vertical component, though the horizontal component is of comparable importance (Back and Bretherton, 2006).
Perhaps the most important distinction across various definitions of vertical GMS is in how vertical motion ($\omega$) profiles are defined. Some definitions make assumptions about the shape of vertical motion profiles. For example, the quasi-equilibrium tropical circulation model (QTCM) is an intermediate-complexity atmospheric model which uses quasi-equilibrium (QE) assumptions to simplify the primitive and energy budget equations. The GMS in the QTCM has an assumed vertical structure ($\text{Neelin and Zeng, 2000}$, $\text{Zeng et al., 2000}$). This type of GMS has relatively little spatial variation and, to the extent that it does, spatial variations are due to variations in thermodynamic profile. However, vertical motion profiles can depart from their QE profiles, so the QTCM could likely be improved by improving the assumptions about vertical motion profile in the GMS calculation.

In order to study convective life cycles, a time-dependent GMS can be compared to a quasi-time-independent GMS in a GMS plane analysis. This GMS plane analysis indicates whether convection is in an amplifying or decaying phase ($\text{Inoue and Back, 2017}$). Here we define GMS on monthly timescales, so here we are making approximations to a quasi-time-independent GMS.

Here we use a definition of vertical GMS which includes spatial variations in vertical velocity and is given by

$$\Gamma_v = \frac{\langle \omega \frac{\partial h}{\partial p} \rangle}{\langle \omega \frac{\partial s}{\partial p} \rangle},$$

(3.1)

where $\Gamma_v$ is the vertical GMS, $\langle \cdot \rangle$ indicates a mass-weighted vertical integral, $\omega$ is vertical pressure velocity, $h = c_pT + gz + Lq$ is MSE, and $s = c_pT + gz$ is DSE. The numerator represents vertically-integrated vertical advection of MSE and the denominator represents vertically-integrated vertical advection of DSE. In order to better visualize the relationship between GMS and horizontal divergence, Equation 1 can be written

$$\Gamma_v = \frac{\langle h \nabla \cdot u \rangle}{\langle s \nabla \cdot u \rangle},$$

(3.2)

where $u$ are the horizontal winds. This is equivalent to Equation 1 by integration by parts and mass continuity. Our definition of GMS is similar to the normalized GMS
of Raymond et al. (2009), but with some differences. Similar to Fuchs and Raymond (2007), we use dry static energy (DSE) advection in the denominator where Raymond et al. (2009) use moisture advection.

The GMS indicates the degree of energy import or export by the circulation. Ascent regions tend to have convergence in the lower troposphere and divergence in the upper troposphere. In that case, positive GMS indicates that the circulation exports energy when the MSE of the diverging upper troposphere is higher than that of the converging lower troposphere. We use the zero contour of vertically-integrated DSE advection to define the boundary between ascent and descent regions, so using DSE advection in the denominator means that the sign of GMS is determined by the numerator.

Despite the prominent role of GMS in theories of the tropical atmosphere, vertical GMS is a difficult quantity to observe because it requires accurate vertical motion and MSE profiles over the atmospheric column. Data from some field campaigns, such as TOGA COARE, DYNAMO, and OTREC, have been used to calculate GMS on limited temporal and spatial scales (e.g. Inoue and Back, 2015b; Sobel et al., 2014). Further, GMS is sensitive to small perturbations in $\omega$ and thermodynamic profiles. With the ubiquity of GMS as a parameter in simple models and the challenges with observing GMS, it is useful to relate GMS to easily-observed quantities. Doing so helps make GMS useable in simple models based on energy budgets, such as the QTCM. Further, simplifications to GMS can build intuition about the opaque metric.

Spatial variations in GMS are strongly influenced by variations in vertical motion profile. In particular, previous work has indicated a close relationship between GMS and top-heaviness ratio, which is a measure of the shape of vertical motion profile (e.g. Inoue et al., 2021). Therefore, improved estimation of vertical motion profiles is a critical step toward estimating and understanding GMS. On monthly timescales, vertical motion is well approximated by its two dominant modes of vertical variability (Back and Bretherton, 2009b). These modes can be rotated without loss of variance

\[ \text{GMS has the same sign as the numerator (MSE advection) in ascent regions and the opposite sign as the numerator in descent regions.} \]
explained to create a shallow mode and a deep mode. The need for both shallow and deep modes to understand geographic variability in GMS is consistent with large variations in top-heaviness ratio across precipitating regions over tropical oceans (Back et al., 2017). Spatial variations in the shallow-mode amplitude can be related to surface convergence (SC) using mass continuity and spatial variations in the deep profile can be related empirically to SST and SC (Back and Bretherton, 2009b; Duffy et al., 2020). Further, SC can be related to the Laplacian of boundary-layer temperature and ultimately to the Laplacian of SST (Lindzen and Nigam, 1987; Back and Bretherton, 2009a; Duffy et al., 2020).

We apply the approximation for monthly vertical motion to vertical motion as it appears in GMS. Combined with the approximation that spatial variations in GMS are dominated by spatial variations in vertical motion, we develop an approximation for GMS that is a function of SST and SC. We then take our approximations a step further to relate GMS to SST and $\nabla^2$SST. These approximations holds in ascent regions over the tropical oceans on monthly timescales. We further evaluate the skill in our approximations in the seasonal cycle in GMS. The paper is organized as follows. Section 2 describes the data used for our analyses. Section 3 derives our GMS approximations. Section 4 discusses the skill of our results on seasonal timescales. Section 5 has the conclusions.

### 3.2 Data

SST data are from NOAA optimal interpolation observations, SC data are from QuikSCAT observations, and all other data are from ERA-Interim reanalysis. We study ocean regions between 20°S to 20°N. All data are interpolated to a 1°x1° horizontal grid and vertical data are interpolated to an evenly-spaced vertical grid with 25 hPa spacing. Monthly data are used in our calculations over the nine year period from 2000-2008, which is the period for which QuikSCAT observations are available. After the Laplacian of SST is taken it is smoothed by convolving with a nine-point averaging filter. SC is smoothed with three passes through the nine-point averaging filter.
filter. The land mask is that of the GFDL-CM3 GCM. The mask is defined such that it extends a bit beyond the land boundary, masking out coastal regions of the oceans.

When plotting, we separate the ascent regions from descent regions. We define the boundary between the two regions where vertically-integrated vertical advection of DSE, the denominator of our GMS definition, is zero. In order to simplify our figures, we plot the vertically-integrated vertical advection of MSE, or the numerator of GMS. Since ascent and descent region are plotted separately and are separated by the zero contour of the denominator, our plotted data have the same sign as GMS in the ascent region and the opposite sign as GMS in the descent region, assuming our approximations do not cause a change in sign in the denominator. Vertical advection of MSE is plotted in Figure 3-1a in the ascent region and Figure 3-1b in the descent region. In the ascent region, vertical MSE advection and GMS are positive in the west pacific and negative in the east pacific, consistent with a deep circulation which exports energy in the west and a shallow circulation which imports in the east. In the descent region, vertical advection is negative nearly everywhere, which corresponds to a positive GMS.

3.3 Relationship between vertical GMS and SST

In order to derive a relationship for $\Gamma_v$ as a function of SST, we focus on making approximations to $\omega$ as it appears in both the numerator and denominator of Equation 3.1. It is well appreciated that, due to weak temperature gradients in the tropics, spatial variations in the vertical advection of DSE are dominated by variations in $\omega$ [Back and Bretherton (2009b)]. More surprisingly, spatial variations in the vertical advection of MSE are dominated by variations in $\omega$, though to a lesser extent than DSE. We demonstrate that this is the case by splitting the MSE lapse rate term into its mean and perturbation terms and as it appears in MSE advection:

$$\langle \omega \partial_p h \rangle = \langle \omega \bar{\partial}_p h \rangle + \langle \omega (\partial_p h)' \rangle,$$  \hspace{1cm} (3.3)
where the overbar indicates a mean over either ascent regions or descent regions and the prime denotes a perturbation from that mean. The three terms in the Equation 3.3 are shown in Figure 3-1 separately for the ascent (left column) and descent (right column) regions. Vertical advection of MSE (Figure 3-1a,b) compares favorably with the spatially-uniform $\partial_p h$ term (Figure 3-1c,d), especially in sign. The difference, the contribution from spatially-varying $\partial_p h$, is shown in Figure 3-1e,f.

By assuming $\partial_p h$ and $\partial_p s$ are spatially uniform, an approximate relationship between GMS and observable quantities can be derived by relating $\omega$ to observable quantities. To do so, we start by applying a two-mode approximation to $\omega$ using empirical orthogonal function (EOF) analysis. Following Back and Bretherton (2009b) and Duffy et al. (2020), we combine the two dominant statistical modes to form a shallow mode and a deep mode. This allows for separate approximations for shallow and deep ascent. We relate the shallow mode amplitude to SC using mass continuity. We relate the deep mode to the shallow mode amplitude, which is proportional to SC, and to SST using thermodynamic arguments. Finally, we relate SC to $\nabla^2$SST using concepts from Lindzen and Nigam (1987) and Duffy et al. (2020).

### 3.3.1 Approximations to $\omega$

We start by applying the two-mode approximation to $\omega$, which separates $\omega$ into deep and shallow modes (Back and Bretherton 2009b; Duffy et al. 2020). Vertical pressure velocity appears in both the numerator and denominator of $\Gamma_v$ and can be approximated using its leading two EOFs, given by

$$\omega(x, y, p, t) \approx \Omega_1(p) o_1(x, y, t) + \Omega_2(p) o_2(x, y, t). \quad (3.4)$$

$\Omega_1$ and $\Omega_2$ are the two structure functions of the EOF analysis that explain the greatest portions of variance in $\omega$ and $o_1$ and $o_2$ are their respective amplitudes. These structure functions are shown in Figure 3-2. Substituting the two-mode approximation for $\omega$ both places it appears in the expression for vertical GMS, Equation 3.1.
Figure 3-1: Vertical MSE advection (a) in the ascent region and (b) in the descent region. Vertical advection of MSE neglecting spatial variations in vertical MSE gradient (c) in the ascent region and (d) in the descent region. Residual (e) in the ascent region and (f) in the descent region.

gives

\[ \Gamma_v \approx \frac{M_{hi} \omega_1 + M_{hi} \omega_2}{M_{si} \omega_1 + M_{si} \omega_2}. \]  

(3.5)

\( M_{hi} = \langle \Omega_i \partial_p h \rangle \) is similar to the GMS of the QTCM, which gives the stratification due to net energy input. As mentioned above, the GMS of the QTCM does not account for spatial variations in \( \omega \) (Neelin and Zeng, 2000). \( M_{si} = \langle \Omega_i \partial_p s \rangle \) is the gross dry stratification, which quantifies the column adiabatic cooling by ascent due to each mode.

The two-mode approximation applied to vertical MSE advection can be seen in Figure 3-3b (ascent) and 3-4b (descent). These can be compared to the unapproximated vertical advection in Figures 3-3a and 3-4a, respectively. Figures 3-3a and 3-4a are identical to Figures 3-1a and 3-1b. The two-mode approximation is quite skillful.

In the first mode, there is strong export of DSE. However, in the second mode, lower-tropospheric export of DSE tends to cancel upper-tropospheric import of DSE. Therefore, \( M_{s1} \omega_1 \) dominates \( M_{s2} \omega_2 \) (Inoue and Back, 2015a). Neglecting \( M_{s2} \omega_2 \), the
Figure 3-2: Vertical motion profiles from EOF analysis (blue) and rotated modes (red).

expression for vertical GMS becomes

\[ \Gamma_v \approx \frac{M_{h1}\omega_1 + M_{h2}\omega_2}{M_{s1}\omega_1} = \frac{M_{h1}}{M_{s1}} + \frac{M_{h2}}{M_{s1}}r, \quad (3.6) \]

where \( r = \frac{\omega_2}{\omega_1} \) is the top-heaviness ratio (Back et al., 2017). \( M_{h1} \) and \( M_{s1} \) are treated as constant in space and time over the ascent and descent regions, separately, consistent with Figure 3-1. Notice that since the \( M_{s1}, M_{s2}, M_{h1}, \) and \( M_{h2} \) are constants, the only spatially-varying dependence of \( \Gamma_v \) is top-heaviness ratio, \( r \). Physically, this means that normalized GMS is primarily dependent upon shape of vertical motion profile. Vertical GMS as approximated using Equation 3.6 is plotted in Figures 3-3c (ascent) and 3-4c (descent). The approximation for vertical advection in Equation 3.6 generally captures the spatial pattern of vertical advection, with some differences in magnitude in the southeast Pacific.

### 3.3.2 Approximating top-heaviness ratio

GMS can now be related to observable quantities, SC and SST, by rotating the leading EOFs \( \Omega_1 \) and \( \Omega_2 \) into a shallow mode, \( \Omega_s \), and deep mode, \( \Omega_d \). The leading EOFs
Figure 3-3: Vertical MSE advection in the ascent region with (a) no approximations, (b) the two-mode approximation, and (c) spatially-uniform $M_h$. 

$\langle \omega \partial_p h \rangle$

$\langle \omega \partial_p h \rangle$, two mode, RMSE = 4.00 W m$^{-1}$

$\langle \omega \partial_p h \rangle$, $M_h$ uniform, RMSE = 10.10 W m$^{-1}$
can be rotated into a shallow and deep mode without loss of variance explained. The shallow mode can then be related to SC and the deep mode can be related to SC and SST as in Back and Bretherton (2009b) and Duffy et al. (2020). The modes are rotated such that the deep mode has zero SC and the shallow mode is orthogonal to the deep mode.

The modes are rotated according to

\[
\begin{align*}
\Omega_s &= -\Omega_1 + c\Omega_2 \\
\Omega_d &= c\Omega_1 + \Omega_2,
\end{align*}
\]  

(3.7)

where \( c = \frac{\Omega_2(975 \text{ hPa}) - \Omega_2(1000 \text{ hPa})}{\Omega_1(1000 \text{ hPa}) - \Omega_1(975 \text{ hPa})} \) because 1000 hPa is the lowest level and 975 hPa is the second lowest level in our analysis. The modes are normalized to have unit length. The shapes of these profiles in monthly ERA-Interim climatologies can be seen in Figure 3-2. The signs of the structure functions are arbitrary. Using Equations 3.7, the mode amplitudes can be written as a function of \( o_s \) and \( o_d \), as given by

\[
\begin{align*}
o_1 &= \alpha_1 o_s + \beta_1 o_d \\
o_2 &= \alpha_2 o_s + \beta_2 o_d,
\end{align*}
\]  

(3.8)

where \( \alpha_1 = \frac{||\Omega_s||}{1-c^2} \), \( \beta_1 = \frac{-c||\Omega_d||}{1-c^2} \), \( \alpha_2 = \frac{c||\Omega_d||}{1+c^2} \), and \( \beta_2 = \frac{||\Omega_d||}{1+c^2} \).

Following Duffy et al. (2020), the shallow and deep mode amplitudes \( o_s \) and \( o_d \) are related to SC and SST. The shallow-mode amplitude \( o_s \) is related to SC using mass continuity according to

\[
o_s = a_s SC.
\]  

(3.9)

where \( a_s \) is given by \( \left( \frac{d\Omega_s}{dp} \right)^{-1}_{\text{surface}} \) which in practice is evaluated using \( \Omega_s \) at the two pressure levels that are nearest the surface.

The deep-mode amplitude \( o_d \) is approximated by a multiple linear regression with relative SST (SST\(_{\text{rel}}\)) and SC,

\[
o_d \simeq b_{\text{SST}} \text{SST}_{\text{rel}} + b_{\text{SC}} SC + b_0,
\]  

(3.10)
where \( b_{SST}, b_{SC}, \) and \( b_0 \) are regression coefficients and \( SST_{rel} \) is defined as SST minus tropical-mean SST for that month. The regression is calculated over tropical oceans. SC is included in the regression to include shallow ascent in the expression for deep ascent: shallow ascent moistens the atmosphere and predisposes it to deep convection. \( SST_{rel} \) is included in the regression since it is related to convective instability (Back and Bretherton, 2009b). The result is insensitive to replacing \( SST_{rel} \) with SST in the regression in Equation 3.10.

At this point, combining Equation 3.6, 3.8, 3.9, and 3.10 gives an approximation for \( \omega \) as a function of SC and SST but we take an additional step to improve the deep-mode approximation by modifying the expression for \( o_d \) if implies negative precipitation. Following Duffy et al. (2020), this approximation to \( \omega \) can be used to approximate precipitation via \( LP \approx \langle \omega \frac{\partial s}{\partial p} \rangle - R \), where \( LP \) is precipitation in energy units and \( R \) is radiative flux convergence. Continuing to neglect spatial variations in DSE advection, plugging in our two-mode approximation to \( \omega \), and replacing \( R \) with its approximation as a function of \( o_s \) and \( o_d \) gives an approximation for precipitation. At this point, we check whether the approximations imply negative precipitation, and, if they do, the deep-mode amplitude is approximated such that the precipitation expression is zero, given by

\[
o_d \approx \frac{R_0 - M_{sed} o_s}{M_{sed}}. \tag{3.11}\n\]

This is equivalent to the use of a Heaviside function in Equation 7 of Duffy et al. (2020).

To get an expression for top-heaviness ratio as a function of SST and SC in regions where Equation 3.11 is not invoked, we combine Equations 3.8, 3.9, and 3.10 to give

\[
\frac{\alpha_2 o_s SC + \beta_2 (b_{SC} SC + b_{SST} SST_{rel} + b_0)}{\alpha_1 o_s SC + \beta_1 (b_{SC} SC + b_{SST} SST_{rel} + b_0)}. \tag{3.12}\n\]

Combining Equation 3.12 with our expression for GMS as a function of top-heaviness ratio, Equation 3.6, results in an expression for vertical GMS as a function of SST.
Figure 3-4: Vertical MSE advection in the descent region with (a) no approximations, (b) the two-mode approximation, and (c) spatially-uniform $M_h$. 
Figure 3-5: Vertical MSE advection in the ascent region with (a) no approximations, (b) GMS as a function of SST and SC, (c) GMS as a function of SST and $\nabla^2 T_v$, and (d) GMS as a function of SST and $\nabla^2$SST.
and SC in regions where Equation 3.11 is not invoked

\[
\Gamma_v = \frac{\bar{M}_{h1}}{M_{s1}} + \frac{\bar{M}_{h2}}{M_{s1}} \left[ \frac{\alpha_2 a_s SC + \beta_2 (b_{SC} SC + b_{SST} SST_{rel} + b_0)}{\alpha_1 a_s SC + \beta_1 (b_{SC} SC + b_{SST} SST_{rel} + b_0)} \right].
\]  

(3.13)

The resulting vertical MSE advection is shown for ascent regions in Figure 3-5b, and compares favorably to unapproximated vertical MSE advection, shown in Figure 3-5a, especially in capturing the sign. Using the above formulation, the approximation is not as skillful in the descent region. In the next section we discuss how our approximation can be modified to improve the skill in the descent region. At this point, we have an expression for GMS which is a function of SST and SC, which could be used in a simple model of tropical energy transport. We further relate SC to low-level temperature after discussing the descent region.

### 3.3.3 Descent region

The approximation for GMS given by Equation 3.13 is better at estimating GMS in the ascent region than in the descent region. Several of the approximations made above are less skillful in the descent region than in the ascent region. For example, neglecting spatial variations in \( \partial_p h \) washes out the gradient in the south Pacific (Figure 3-1). Further, the two-mode approximation at the surface is poor in the descent region. Consequently, the relationship between shallow-mode amplitude and SC, Equation 3.9, which carries over into the deep mode amplitude approximation, Equation 3.10, is problematic.

An effective but inelegant solution is to modify the shallow-mode approximation to use convergence at 950 hPa in place of SC. In the ascent region, we use SC from QuikSCAT observations but here we use convergence at 950 hPa from ERA-Interim reanalysis, denoted \( \nabla \cdot U_{950} \). The shallow-mode approximation now comes from

\[
\nabla \cdot U_{950} = \left( \frac{\partial \omega}{\partial p} \right)_{950} \approx \left( \frac{\partial \Omega_s}{\partial p} \right)_{950} o_s + \left( \frac{\partial \Omega_d}{\partial p} \right)_{950} o_d. \]

(3.14)

Looking at Figure 3-2, it is clear that the \( \Omega_d \) profile is relatively constant near 950
hPa (i.e. \( \frac{\partial \Omega}{\partial p} \). Therefore, we have

\[
o_s \approx a^{950}_s \nabla \cdot \mathbf{U}_{950} \quad \text{(descent),} \tag{3.15}
\]

where \( a^{950}_s = \left( \frac{\partial \Omega}{\partial p} \right)^{-1}_{950} \). Further, the deep mode regression given by Equation 3.10 includes SC, the shallow-mode amplitude, because the shallow ascent predisposes the atmosphere to deep ascent. Thus, in order to better constrain the deep mode in the descent region, we replace SC with \( \left( \frac{\partial \Omega}{\partial p} \right)_{950} \) in the regression, and limit the regression to the descent region. Therefore, the approximation for \( o_d \) is the descent region is given by

\[
o_d \approx b^d_{\text{SST}} \text{SST}_{\text{rel}} + b^d_{\text{SC}} \nabla \cdot \mathbf{U}_{950} + b^d_0, \tag{3.16}
\]

where the \( d \) superscript indicates that the regression coefficient is calculated using data from the descent region alone. The resulting approximation for vertical MSE advection is shown in Figure 3-6b. We do not expect \textit{a priori} that our approximation will be as skillful in the descent region as it is in the ascent region because the deep mode approximation is designed with ascent and convection in mind. Alas, our estimate of vertical advection compares favorably with vertical advection in sign, but differs in magnitude in many locations. We do not further relate \( \nabla \cdot \mathbf{U}_{950} \) to temperature gradients as we do in the ascent region.

### 3.3.4 Approximations to SC

Generally, SC can be related to temperature via pressure gradients: SC is the convergence of the surface wind field, which is partially set by pressure gradients. These pressure gradients are largely proportional to temperature gradients by hydrostatic balance (Lindzen and Nigam, 1987; Back and Bretherton, 2009a). More specifically, following Duffy et al. (2020), SC, which appears in both the shallow- and deep-mode amplitude approximations can be related to (1) the Laplacian of boundary-layer virtual temperature \( T_v \), and (2) the Laplacian of SST. The expression for SC as a function of the Laplacian of boundary-layer virtual temperature \( \nabla^2 T_v \) is derived from a mixed-layer model of the boundary layer that represents momentum conserva-
Figure 3-6: Vertical MSE advection in the descent region with (a) no approximations and (b) GMS as a function of SST and $\nabla \cdot \mathbf{U}_{950}$. It is given by

\[
SC(\nabla^2 T_v) \simeq -\frac{P_s g \epsilon_i}{R_d \rho_0 (\epsilon_i^2 + f^2)} \int_{BL} \frac{\nabla^2 T_v}{T_v^2} \, dz,
\]

where $P_s$ is surface pressure, $g$ is acceleration due to gravity, $\epsilon_i = 7 \times 10^{-5}$ s$^{-1}$ is a frictional coefficient from Duffy et al. (2020), $R_d$ is the gas constant for dry air, $\rho_0$ is a constant reference density, $f$ is the Coriolis parameter, virtual temperature is given by $T_v = T(1 + 0.61q)$ where $q$ is specific humidity and $T$ is temperature, $\nabla^2$ is the Laplacian operator, and $\int_{BL} \, dz$ is the integral over the boundary layer which in practice is evaluated in pressure coordinates between 1000hPa and 850hPa. SC can be replaced with $SC(\nabla^2 T_v)$ as it appears in the expression for GMS, Equation 3.13.

Following Duffy et al. (2020), the expression for $SC(\nabla^2 T_v)$ can be further approximated to give SC as a function of SST by assuming that the pattern of SST decays through the boundary layer by a factor of 2. The factor of 2 decay represents the fact the temperature pattern imprinted on the atmosphere by SST decays with height.
Further, we replace $T_v^2$ in the denominator with $SST^2$ because the magnitude of $T_v$ is close to that of SST, both of which have units of K. The resulting expression for SC as a function of SST is

$$SC(\nabla^2 SST) \simeq -\frac{3\bar{z}_{850} p_s g \epsilon_i}{4 R_d \rho_0 (\epsilon_i^2 + f^2) SST^2} \nabla^2 SST,$$

where $\bar{z}_{850}$ is the spatial-mean height of the 850 hPa surface. SC can be replaced with $SC(\nabla^2 SST)$ as it appears in the expression for GMS, Equation 3.13.

The vertical MSE advection as a function of SST and $SC(\nabla^2 T_v)$ is shown in Figure 3-5c and the vertical GMS as a function of SST and $SC(\nabla^2 SST)$ is shown in Figure 3-5d. While there are some regions of disagreement (e.g. the southern portion of the west Pacific for both sets of approximations), both approximations broadly capture vertical MSE advection, especially the sign.

### 3.4 Discussion

#### 3.4.1 Seasonal cycle

Seasonal variations in GMS may play an important role in understanding the seasonal cycle of tropical atmospheric behavior. For example, GMS may be important in modulating ITCZ shifts on seasonal and sub-seasonal timescales. In particular, the energy flux equator (EFE) has become a popular metric for location of the ITCZ, but there may be a lag in the seasonal migration of ITCZ as compared with EFE; a negative GMS may explain the lag (Wei and Bordoni 2018). We plot the difference in vertical MSE advection in the region which has climatological ascent during both DJF and JJA between boreal summer and boreal winter in Figure 3-7a. The seasonal cycle of MSE advection can be compared to its approximation as a function of SST and SC in Figure 3-7b and as a function of SST and $\nabla^2 SST$ in Figure 3-7c. The vertical MSE advection tends to be higher in the summer hemisphere. That is, JJA-DJF of MSE advection tends to be positive in the northern hemisphere and negative in the southern hemisphere. The higher MSE advection may not correspond to higher GMS.
Figure 3-7: Seasonal cycle of vertical MSE advection in the region that has climatological ascent in both DJF and JJA with (a) no approximations, (b) GMS as a function of SST and SC, and (c) as a function of SST and $\nabla^2\text{SST}$.

because of stronger ascent in the summer hemisphere. Nonetheless, the difference in MSE advection across the seasonal cycle is generally captured by both sets of approximations for SC. We further evaluate seasonal variations in MSE advection in the descent region. Figure 3-8 reveals that our approximation for MSE advection is less skillful in the descent region than in the ascent region when evaluating the seasonal cycle, especially south of the equator.
3.4.2 Applications

As mentioned in the introduction, the approximations used in deriving $\Gamma_v(SST, \nabla^2 SST)$ also have potential applications to the QTCM, an intermediate complexity atmospheric model that is especially relevant to convecting regions in the tropics and their interactions with the large-scale circulation. The QTCM defines GMS by assuming a shape of the vertical motion profile. The shape of $\omega$ cannot account for variations in top-heaviness, so the QTCM could potentially be improved by allowing for spatial variations in $\omega$.

3.5 Conclusions

We have developed an approximation for vertical GMS as a function of SST and its Laplacian which captures spatial variations in GMS in precipitating regions over tropical oceans on monthly timescales. Whereas calculating the vertical GMS requires knowledge of $\omega$ and $h$ profiles, approximating it using $\Gamma_v(SST, \nabla^2 SST)$ requires only
the SST field.

We have made substantial approximations to GMS. In particular, we have neglected spatial variations in the $M_{hi}$. Our GMS expression could potentially be improved by relating $M_{hi}$ to SST. A limitation of our method is the use of a single reanalysis dataset. It is known that there are variations across reanalysis products, so our results may be sensitive to our choice of dataset [Back and Bretherton, 2009b].

A pressing area of future work is understanding the horizontal advection terms, which are known to be of comparable importance to vertical advection terms in the MSE budget. A complete picture of the atmospheric MSE budget relies on an understanding of both horizontal and vertical advection terms. Our approximations may be applicable to a portion of the horizontal advection of moisture [Raymond and Zeng, 2005]. In the next chapter, we develop a new definition of GMS, the boundary GMS, which approximately incorporates both vertical and horizontal GMS and is relevant for idealized simulations with an imposed surface heat flux. It is plausible that approximations made to GMS in this chapter are applicable to the boundary GMS.
Chapter 4

An energetic evaluation of the response of the Walker circulation to warming

Abstract

There is uncertainty about the sign and magnitude of the response of the Walker circulation (WC) to warming. Proposed atmospheric mechanisms for the response of the WC to warming suggest a weakening. Trends in observations are mixed, with longer-term trends indicating a weakening and shorter-term trends indicating a strengthening. Model projections typically simulate a weakening with warming, but with substantial spread in the degree of weakening. Here we diagnose contributions to WC response in CMIP5 and AMIP model projections using a moist static energy (MSE) budget. In each model, we evaluate the contributions to changes in the WC strength due to changes in gross moist stability (GMS), horizontal MSE advection, radiation, and surface latent and sensible heat fluxes. We find that the multimodel mean weakening is due mostly to changes in GMS and radiation. Further, we find that the spread in the WC response is substantial across both the CMIP5 and AMIP models, implying that differences in atmospheric models are important for the spread in projected WC strength. Changes in GMS can explain a substantial portion of both the mean and spread in WC responses. Parameterized convective entrainment can affect the GMS response through changes in depth of convection and through changes in lapse rates. We evaluate the role entrainment in setting the GMS response by varying the entrainment rate in an idealized GCM. The idealized GCM is run with a simplified Betts-Miller convection scheme, modified to represent entrainment. The idealized GCM simulates a weakening of the WC with warming, and this weakening is dampened by increasing entrainment. However, the spread in GMS responses
due to differing entrainment rates is smaller than the spread in GMS response across CMIP5 models. Therefore, further work is needed to evaluate the large spread in GMS responses across CMIP5 models.

4.1 Introduction

The Pacific Walker circulation (WC) is an atmospheric zonal circulation over the equatorial Pacific Ocean. The WC transports energy from the west Pacific to east Pacific in response to differing sea surface temperatures (SST) over this region. This is associated with a zonal surface pressure gradient over the Pacific ocean, whose interannual variability comprises the southern oscillation. The response of the WC to a warming climate has been assessed using a combination of theory, observations, historical model trends, and model projections. Together, these lines of evidence give an unclear picture of the response of the WC to warming.

There are a number of proposed mechanisms for the response of the WC to warming, some of which suggest a weakening and some of which suggest a strengthening. Tropical convective mass fluxes must weaken overall with warming because precipitation increases lag the increase in specific humidity with warming, which is set by the Clausius-Clapeyron relationship (Held and Soden, 2006). Knutson and Manabe (1995) found a weakening of the WC in projections despite an increase in precipitation. Increases in dry static stability, which are the result of changes in moist adiabatic lapse rate, have been implicated in this weakening (Knutson and Manabe, 1995). Further, differential increases in evaporative damping between the warm west Pacific and cool east Pacific weaken the SST gradient (Knutson and Manabe, 1995; Ma et al., 2012). Additionally cloud masking, a direct CO$_2$ effect, contributes a weakening of the WC (Merlis, 2015).

In contrast, an ocean dynamical thermostat mechanism and changes in anthropogenic aerosols may contribute a strengthening of the zonal SST gradient with warming (Clement et al., 1996; Heede and Fedorov, 2021). The ocean dynamical thermostat mechanism, which was proposed using a highly idealized ocean model, strengthens the zonal SST gradient with two processes. First, upwelling of cool water in the equa-
torial east Pacific ocean reduces eastern Pacific SSTs, thereby increasing the zonal gradient. Second, this gradient is further maintained by the resulting increase in surface easterly winds (Clement et al., 1996). An analysis of coupled GCMs found the upwelling portion of the mechanism to be operating but not the atmospheric portion of the mechanism because the easterly winds tend to weaken in the models. The net effect is a slight weakening of the zonal SST gradient in GCMs (DiNezio et al., 2009). Further, evidence from CMIP6 models suggests an overall cooling of the surface in response to changes in aerosols with an outsized cooling of the equatorial east Pacific (Heede and Fedorov, 2021).

Observational and reanalysis products going back only a few decades indicate a strengthening of the WC while observations over the past century indicate a weakening (Vecchi et al., 2006; Sohn et al., 2016). A large role of internal variability suggests that long time periods are needed to evaluate trends in the WC, which may explain the discrepancy (Vecchi et al., 2006). Coupled climate model trends over the historical period of WC strengthening are mixed, with some models indicating a weakening and others indicating a strengthening, though no model strengthens to the same extent as observations (Sohn et al., 2016). Projections of a warm 21st century climate almost unanimously indicate a WC weakening, but with substantial spread in the degree of weakening (Vecchi and Soden, 2007). Here we seek to understand the spread in WC response across GCM projections.

The WC response to warming varies inversely with the increase in GMS with warming in simulations with an idealized GCM, consistent with an analysis of the MSE budget (Wills et al., 2017). Wills et al. (2017) use a definition of GMS which is relevant to the WC with zonally-anomalous vertical advection in the numerator and the strength of the WC in the denominator. GMS measures the efficiency of a circulation in exporting energy. For a given zonal gradient of net energetic input to the atmosphere we expect an increase in GMS with warming to correspond to a weaker WC and a decrease in GMS with warming to correspond to a stronger WC (Wills et al., 2017). In general, we expect the GMS to increase with warming owing predominantly to an increase in tropopause height (Chou et al., 2013). In
the real atmosphere and in more realistic simulations, we expect a more complicated relationship between GMS and WC responses than in the idealized simulations of (Wills et al., 2017). Nonetheless, we also find an inverse relationship between WC response and changes in GMS in CMIP5 and AMIP models. The close relationship between responses of WC strength and GMS across CMIP5 models, AMIP models, and an idealized GCM warrants further investigation into the response of GMS to warming. We focus on the role of convective entrainment in setting the response of the WC and GMS.

In general, entrainment is the process by which a buoyant plume mixes with the environment. Increasing entrainment tends to suppress convection and steepen the temperature lapse rate (Held et al., 2007). Changes in both lapse rate and depth of convection can affect the GMS (Held et al., 2007). However, it is difficult to represent entrainment in GCMs. Entrainment occurs on subgrid scales and entrainment rates are difficult to measure directly (Romps, 2010). Following Wills et al. (2017), we use an idealized GCM with a simplified Betts-Miller (SBM) convection scheme to evaluate the role of entrainment in the WC and GMS response across climates. To do so, we modify the SBM scheme to represent entrainment.

This chapter has two aims: (1) diagnose the contributions to the mean and spread of the WC response to warming in CMIP5 and AMIP models, and (2) evaluate the influence of entrainment on WC strength and its response to warming in an idealized GCM. We address the first aim in Section 2. We address the second aim in Section 3. We discuss and conclude in Section 4.

4.2 Response of Walker circulation to warming in CMIP5 and AMIP simulations

4.2.1 WC decomposition using GMS and MSE - theory

We diagnose the contributions to the response of the Walker circulation to warming across CMIP5 and AMIP models. We use monthly data of each variable and take
climatologies of calculated terms before taking the difference between warm and control climates. For the CMIP5 simulations, ‘control climate’ refers to the historical simulations for the years 1980 to 1999 and ‘warm climate’ refers to the RCP8.5 simulations for the years 2080 to 2099. For the AMIP simulations, ‘control climate’ refers to the historical simulation for the years 1979 to 2008 and ‘warm climate’ refers to the AMIP future simulations for the years 1979 to 2008. The AMIP simulations are atmosphere-only simulations run with the same imposed SST field across models. The imposed SST field of the historical simulations is from observations. The imposed SST field of the future simulations is of realistic future warming, including a change in pattern. Throughout this chapter, tropical-mean warming is used to normalize (e.g. to calculate % K$^{-1}$).

We develop a framework for diagnosing contributions to changes in WC strength by combining the GMS with the MSE budget. The WC strength is given by $\overline{\omega_w - \omega_e}$, where $\omega$ is vertical velocity in pressure coordinates, the overbar indicates a vertical average in pressure coordinates, and $w - e$ denotes a difference between the average over a western Pacific box and an eastern Pacific box. Both boxes extend from $10^\circ$S to $10^\circ$N. The western Pacific box extends from $80^\circ$E to $160^\circ$E and the eastern Pacific box extends from $160^\circ$W to $80^\circ$W. The western Pacific box includes a small portion of the Indian ocean.

The time-average moist static energy budget is given by

$$\left\langle \omega \frac{\partial h}{\partial p} \right\rangle_{w-e} = -\langle u \cdot \nabla h \rangle_{w-e} + R_{w-e} + Q_{w-e},$$

where $h = c_p T + gz + Lq$ is moist static energy, $c_p$ is the heat capacity of dry air, $T$ is temperature, $g$ is acceleration due to gravity, $z$ is height, $L$ is latent heat of vaporization, and $q$ is specific humidity. $\langle \cdot \rangle$ indicates a mass-weighted vertical integral, $u$ are horizontal winds, $Q$ is the sum of upward surface fluxes of latent and sensible heat, and $R$ is net longwave and shortwave radiative fluxes into the atmosphere. All four terms in Equation 4.1 are time averages.

A definition of GMS appropriate for the WC is used here. The GMS is the ratio of
vertical advection of MSE, differenced between the western and eastern Pacific boxes, to the WC strength and is given by

$$\text{GMS} \equiv -g \left\langle \frac{\omega \partial h}{\partial p} \right\rangle_{w-e} = -g \left\langle \frac{\partial h}{\partial p} \right\rangle_{w-e},$$  \hspace{1cm} (4.2)$$

where $g$ is acceleration due to gravity and $\bar{\omega} = \frac{\omega}{\text{GMS}_{w-e}}$ is the shape of the vertical-velocity profile. The denominator is equivalent to the negative of the WC strength. This definition of GMS is similar to that of [Wills et al., 2017] with two differences. First, instead of taking a zonal anomaly, we take the difference between the western and eastern Pacific boxes, denoted with subscript $w - e$. Further, we have modified the definition of WC strength, which appears in the denominator. [Wills et al., 2017] defined the WC strength by the zonally-anomalous vertical velocity at the level of its maximum, $\omega^*_{max}$. Instead, we use vertically-averaged $\omega$ and the difference between the western and eastern Pacific boxes, denoted $\overline{\omega}_{w-e}$.

In order to derive a diagnostic expression for WC strength from the MSE budget, we combine Equations 4.1 and 4.2 to give

$$\overline{\omega}_{w-e} = -g \left\langle u \cdot \nabla h \right\rangle_{w-e} + R_{w-e} + Q_{w-e} \overline{\text{GMS}}.$$  \hspace{1cm} (4.3)$$

We further decompose the radiation contribution into a contribution from the WC and a residual contribution using the following regression

$$R_{w-e} \approx r_1 \overline{\omega}_{w-e} + R_0.$$  \hspace{1cm} (4.4)$$

The regression is applied across the seasonal cycle to climatologies of $R_{w-e}$ and $\overline{\omega}_{w-e}$ in each model for each climate. The $r_1 \overline{\omega}_{w-e}$ term is interpreted as the contribution to $R_{w-e}$ which is linked with WC strength and $R_0$ is interpreted as the contribution to $R_{w-e}$ which is not linked with WC strength. The GMS and the $r_1 \overline{\omega}_{w-e}$ terms combine to form an effective GMS similar to that of e.g. [Fuchs and Raymond, 2007].

Finally, considering a perturbation due to climate change gives an expression for the response of the WC to warming as a function of changes in GMS, horizontal MSE.
advection, surface heat fluxes, and radiation. This expression is given by

\[
\delta \omega_{w-e} = -\delta \text{GMS} - \frac{\Delta (u \cdot \nabla h)_{w-e}}{\langle \omega \frac{\partial h}{\partial p} \rangle_{w-e}} + \frac{\Delta R_{w-e}}{\langle \omega \frac{\partial h}{\partial p} \rangle_{w-e}} + \frac{\Delta Q_{w-e}}{\langle \omega \frac{\partial h}{\partial p} \rangle_{w-e}} + \text{resid}, \tag{4.5}
\]

where \( \Delta \) indicates a response to warming, \( \delta \) is the fractional response to warming given by \( \delta X = \frac{\Delta X}{X} \), and \( X \) is the average between the control and warm climates. The terms on the RHS of Equation 4.5 are the contributions to the WC response from changes in GMS, horizontal advection, radiation, surface heat fluxes, and a residual, respectively. The residual includes errors due to finite differencing in calculating vertical advection and errors in energy conservation. We replace the radiation term in Equation 4.5 with its approximation in Equation 4.4

\[
\delta \overline{\omega}_{w-e} = -\delta \text{GMS} - \frac{\Delta (u \cdot \nabla h)_{w-e}}{\langle \omega \frac{\partial h}{\partial p} \rangle_{w-e}} + \frac{\Delta (r_{1} \omega_{w-e})}{\langle \omega \frac{\partial h}{\partial p} \rangle_{w-e}} + \frac{\Delta R_{0}}{\langle \omega \frac{\partial h}{\partial p} \rangle_{w-e}} + \frac{\Delta Q_{w-e}}{\langle \omega \frac{\partial h}{\partial p} \rangle_{w-e}} + \text{resid}. \tag{4.6}
\]

We apply Equation 4.6 to each CMIP5 and AMIP model.

### 4.2.2 WC response decomposition in CMIP5

In order to diagnose contributions to changes in WC strength in coupled GCMs, we apply the decomposition given by Equation 4.6 to each CMIP5 model. Figure 4-1 shows the decomposition in the multimodel mean (left) and in individual GCMs (right). We find that the WC weakens in all but the inmcm4 model, with a range of a 18.8% \( \text{K}^{-1} \) weakening to 1.3% \( \text{K}^{-1} \) strengthening. The multimodel mean of a 9% \( \text{K}^{-1} \) weakening falls within the 5 to 10% \( \text{K}^{-1} \) estimated by Vecchi and Soden (2007) using changes in \( \omega_{500} \).

Looking at Figure 4-1, we notice that the relative roles of each mechanism in setting the WC response can vary substantially across models, but a few noteworthy commonalities emerge clearly in the multimodel mean. First, the total radiation contribution, which is well approximated by the sum of the WC-linked and not WC-linked potions, and the WC-linked radiation contribution are both the same sign as the WC response in all CMIP5 models. That is, changes in radiation and the
WC-linked portion of radiation contribute a weakening in all but the inmcm4 model. We hypothesize that this is due to a robust amplifying feedback of radiation on WC response across models (Peters and Bretherton 2005). Second, the response of GMS consistently contributes a weakening of the WC across models. That is, GMS increases with warming in all models, consistent with Chou et al. (2013). The contribution from changes in radiation range from a weakening of 10.1% K$^{-1}$ to a strengthening of 1.6% K$^{-1}$, with the WC-linked portion dominating in the multimodel mean. The contribution from changes in GMS range from a weakening of 15.5 to 2.2% K$^{-1}$.

### 4.2.3 WC response decomposition in AMIP

In order to isolate the atmospheric contribution to the spread in WC response, we compare the response of the WC in AMIP simulations between the ‘AMIP historical’ and the ‘AMIP future’ simulations. All of the AMIP historical simulations have the same imposed SST distribution as one another and all of the AMIP future simulations have the same imposed warmer SST distribution as one another.

Figure 4-2 is the same as Figure 4-1 except with AMIP models instead of CMIP5 models. Even with the same SST response across models, there is spread in the response of the WC of 18.5 to 5.1% K$^{-1}$ weakening. While the substantial spread in WC response across AMIP models does not rule out an important role of the ocean in setting the spread, it does indicate an important role of the atmosphere in setting the spread. Similar to the CMIP5 analysis, the contribution from changes in GMS dominates the WC response. However, the multimodel-mean contributions from horizontal advection and surface fluxes are more substantial in the AMIP models than in the CMIP5 models. Further, unlike the CMIP5 models, in the multimodel mean the radiation contribution has equal contributions from the WC-linked portion and the other portion. We do not further investigate these differences between AMIP and CMIP5 analyses.
Figure 4-1: Contributions to response of WC to warming in CMIP5 models in the multimodel mean (left) and in individual models (right). Each bar corresponds to a term in Equation 4.5. The whiskers on the multimodel mean cover the spread in individual model contributions.

Figure 4-2: Same as Figure 4-1 except in AMIP models.
4.2.4 Relationship between WC and GMS responses

Figures 4-3a and 4-3b show the relationship between responses of WC strength and GMS in CMIP5 and AMIP models. The correlation coefficient is -0.84 across the CMIP5 models and -0.79 across the AMIP models. Wills et al. (2017) showed a similar relationship between WC strength and GMS in idealized simulations. The high correlation between responses of WC strength and GMS indicates that the WC-GMS relationship holds in more realistic simulations and warrants further investigation into the response of GMS to warming. In order to better understand the response of GMS to warming in CMIP5 and AMIP models, we next decompose GMS response into its contributions due to changes in MSE profiles and changes in shape of vertical motion profile.

4.2.5 GMS decomposition

We expect GMS to vary inversely with WC strength because a larger increase in GMS indicates a weaker response of the atmospheric circulation for a given energetic forcing. Figure 4-4 shows the spread in WC strength and GMS responses to warming in CMIP5.
models. The models are sorted from most weakening (left) to least weakening (right). The response of GMS to warming is robustly positive across models. Looking at Equation 4.2, the fractional change in GMS with warming has contributions from changes in $\tilde{\omega}$ and changes in $h$ profile as follows

$$\delta \text{GMS} = \frac{\langle \Delta \tilde{\omega} \frac{\partial h}{\partial p} \rangle_{w-e}}{\langle \tilde{\omega} \frac{\partial h}{\partial p} \rangle_{w-e}} + \frac{\langle \tilde{\omega} \Delta \frac{\partial h}{\partial p} \rangle_{w-e}}{\langle \tilde{\omega} \frac{\partial h}{\partial p} \rangle_{w-e}}.$$  

(4.7)

As above, $\delta$ is the fractional response to warming given by $\delta X = \frac{\Delta X}{X}$ where $X$ is the average between the warm and control climate and $\Delta X$ is the response to warming. We find that the $\tilde{\omega}$ contribution tends to be larger in magnitude than the $h$ profile contribution. The spread in the increase in GMS with warming in CMIP5 models ranges from 2.2 to 15.5% K$^{-1}$. The $\tilde{\omega}$ contribution to GMS ranges ranges from 3.6 to 14.2% K$^{-1}$ across models. The $h$ contribution to GMS ranges from -6.3 to 0.9% K$^{-1}$. We expect a dominant mechanism of GMS response to climate warming to be a weakening due to increase in tropopause height and associated upward shift of $\tilde{\omega}$, resulting in an increased GMS (Singh and O’Gorman, 2012; Chou et al., 2013). This expected upward shift could explain the substantial contribution of changes in $\tilde{\omega}$ to changes in GMS.

The $h$ profile contribution can be further decomposed into contributions from temperature, geopotential height and specific humidity. Further, the changes in specific humidity can be decomposed into its contributions from changes in saturation specific humidity and relative humidity, according to $\Delta q = \Delta RHq_{sat} + RH\Delta q_{sat}$. Looking at Figure 4-4, changes in $h$ have a small contribution in several models, but are the result of compensation between strong positive contributions from changes in $T$ and $\Phi$ and a strong negative contribution from changes in humidity. These changes in humidity, which act to decrease the GMS, are mostly the result of changes in saturation specific humidity.

Spread in GMS comes from spread in both MSE profiles and vertical motion profiles. In the next section, we use an idealized GCM to look at the role of entrainment in the response to GMS.
Figure 4-4: Contributions to response of GMS to warming in CMIP5 models in the multimodel mean (left) and in individual models (right). GMS is given by Equation 4.2. Its response to warming (black) is decomposed into contributions from changes in shape of vertical velocity profile (dark red) and changes in MSE (light red). The MSE contribution (light red) is further decomposed into contributions from changes in temperature (dark green), geopotential height (medium green), and humidity (light green). The humidity contribution (light green) is further decomposed into contributions from changes in saturation specific humidity (dark purple) and relative humidity (light purple). The whiskers on the multimodel mean cover the spread in individual model contributions.
4.3 The role of entrainment in idealized GCM simulations

4.3.1 Why consider entrainment?

In order to further evaluate the spread in WC strength responses, we study the role of entrainment in setting the WC strength and its response to warming in an idealized GCM. Entrainment is a parameterized process which is difficult to quantify in observations. However, entrainment parameterizations can have a substantial effect on the simulated climate, especially in the tropics (Singh and O’Gorman, 2013; Miyawaki et al., 2020). Entrainment affects the temperature lapse rate: a higher entrainment rate tends to steepen the temperature profile (Held et al., 2007). Further, variations in temperature with entrainment mean changes in specific humidity with entrainment. The temperature and humidity profiles influence the MSE profile, a key portion of the GMS. Further, entrainment can influence the depth of convection and thereby affect the shape of the vertical motion profile, which is strongly influences the GMS, as shown in Chapter 3. Further, the representation of entrainment is parameterized, with differences in parameterization across GCMs.

4.3.2 Idealized GCM simulations

Idealized simulations of the Walker circulation are run using an idealized moist atmospheric GCM with the GFDL dynamical core (Frierson et al., 2006; O’Gorman and Schneider, 2008). The idealized GCM lacks land, a seasonal cycle, cloud-radiative effects, and water-vapor radiative feedbacks. The model has a thermodynamic mixed-layer ocean with a depth of 1m. There is a zonal-mean Q flux with a maximum amplitude of 50 W m$^{-2}$. Following Wills et al. (2017), the WC is driven by an imposed zonally-anomalous Q flux with an elliptic ascent region in the ‘western’ hemisphere and an equal and opposite descent region in the ‘eastern’ hemisphere, both centered
Figure 4-5: Imposed zonally-anomalous $Q$ flux of the idealized GCM simulations.

on the equator. The zonally-anomalous $Q$ flux has the form

$$ Q^* = Q_1 \exp \left[ \frac{(\lambda - \lambda_E)^2}{2\sigma_\lambda^2} + \frac{\phi^2}{2\sigma_\phi^2} \right] - Q_1 \exp \left[ \frac{(\lambda - \lambda_W)^2}{2\sigma_\lambda^2} + \frac{\phi^2}{2\sigma_\phi^2} \right], $$  \hspace{1cm} (4.8)

where $\lambda$ is longitude, $\phi$ is latitude, $Q_1 = 50$ W m$^{-2}$ is the amplitude of the zonally-anomalous $Q$ flux, $\lambda_E = 270^\circ$ is the longitude of the center of the descent region, $\lambda_W = 90^\circ$ is the longitude of the center of the ascent region, $\sigma_\lambda = 12.5^\circ$ is proportional to the zonal extent of the anomaly, and $\sigma_\phi = 8^\circ$ is proportional to the meridional extent of the anomaly. The sign of the zonally-anomalous $Q$ flux is modified from Wills et al. (2017) such that positive indicates a flux from ocean to atmosphere. The imposed zonally-anomalous $Q$ flux is plotted in Figure 4-5.

The idealized simulations are spun up for 4 years and the analysis is performed on the following 8 years of simulation output. The convection scheme is a modification of the simplified Betts-Miller (SBM) convection scheme, which relaxes temperature profiles to a moist adiabat and relative humidity to 70% in convecting regions (Frierson, 2007). Here, we modify the SBM scheme by introducing an entrainment parameter such that the convection scheme relaxes to an *entraining* adiabat when the entrainment parameter is greater than 0. Our entraining SBM scheme reduces to the SBM convection scheme when the entrainment parameter is 0. Details about the modifi-
cation to represent entrainment are given in Appendix C.

Two climates are simulated: a control climate with a default longwave optical depth ($\alpha = 1$) and a warm climate with doubled longwave optical depth ($\alpha = 2$), where $\alpha$ is a longwave optical depth scaling factor. The difference between the two climate simulates a climate warming with a global-mean SST increase of 11.2K and a tropical-mean increase of 9.1K in the simulations without entrainment.\footnote{Interestingly, global-mean warming increases from 11.2K when $\epsilon = 0$ to 12.1K when $\epsilon = 1/2$. Tropical-mean warming increases from 9.1K when $\epsilon = 0$ to 10.2K when $\epsilon = 1/2$.} We run the idealized model for simulations for a control climate and a warm climate with four entrainment parameters each, for a total of 8 simulations. The four nondimensional entrainment parameters are 0 (no entrainment), 1/8, 1/4, and 1/2.

### 4.3.3 Spread in vertical MSE profiles

GMS depends on both the shape of the vertical motion profile and the MSE profile. To visualize the effect of entrainment on MSE profile, we plot changes in MSE profiles from idealized simulations across entrainment parameters and compare them to MSE profiles from CMIP5 and AMIP models. Figure 4-6 compares the response of MSE profiles to warming in CMIP5 models (left), AMIP models (center), and across entrainment rates in the idealized GCM (right). The response of surface MSE is subtracted from each profile so that the profiles all go through 0 at the surface. For the CMIP5 and AMIP models, MSE profiles are averaged over the area of the western Pacific box. For the idealized models, profiles are averaged over the boundary of the elliptic ascent region, consistent with the GMS analysis in the next section. The boundaries are defined by the $\pm 10$ W m$^{-2}$ contours which can be seen in Figure 4-5. We find that the spread in MSE profile responses across entrainment rates in the idealized GCM is similar to the spread in MSE profile responses across CMIP5 and AMIP models.
4.3.4 Sensitivity of WC strength to warming and entrainment in idealized simulations

We evaluate the WC strength in 8 simulations using the idealized GCM. The WC strength, defined as the negative of the average value of $\omega$ in the ascent region minus the average value of $\omega$ over the descent region, is shown in Figure 4-7. In general, the WC is weaker in the warm climate than in the cool climate. For a given climate, WC strength increases with increasing entrainment. We hypothesize that the sensitivity to entrainment is higher in the warm climate because the degree of MSE subsaturation, $h_s - h_e$, is larger in the warm climate. As a result, WC weakens with warming more at lower entrainment rates than it does at higher entrainment rates. That is, the difference between the red and blue lines decreases with entrainment in Figure 4-7. While entrainment does affect the response of the WC to warming, the spread due to variations in entrainment of 1.6 % K$^{-1}$ is not as large as the spread due to differences across models in CMIP5 (20.0% K$^{-1}$) or AMIP (13.5 % K$^{-1}$). However,
radiative feedbacks are not as fully represented in the idealized simulations as they are in the CMIP5 and AMIP simulations, and our analysis suggests that they have an amplifying effect on the WC response.

4.3.5 Gross Moist Stability in idealized simulations

From Wills et al. (2017), the Walker circulation strength varies inversely with the zonally-anomalous GMS in this idealized set up with an entrainment parameter of 0. Does this relationship between WC strength and GMS hold with variations in entrainment? Looking at Equation 4.5, we notice that in the idealized simulations, $\Delta Q_{w-e} = 0$ because our imposed Q flux in Equation 4.8 does not change across simulations. Further, in the idealized simulations, $\Delta R_{w-e} \approx 0$ because the simulations do not have cloud-radiative effects or water vapor-radiative feedbacks, so the radiation is very nearly zonally uniform. Consequently, in the idealized simulations, Equation 4.5 reduces to

$$\delta \omega_{w-e} \simeq -\delta \text{GMS} - \frac{\Delta \langle u \cdot \nabla h \rangle_{w-e}}{\langle \omega \frac{\partial h}{\partial p} \rangle_{w-e}} \text{ (idealized)},$$

(4.9)

where $\delta$ is a fractional response and $\Delta$ is a difference between simulations. Thus, there is an inverse relationship between WC strength response and GMS response if changes in horizontal advection terms are small. However, we are not aware of a theory for the relationship between horizontal advection and entrainment, and there is no reason a priori to expect the response of horizontal advection to be small.

To evaluate the role of horizontal advection, we compare WC strength with the inverse of GMS. Figure 4-8 shows that GMS response is not closely correlated with WC response, indicating that changes in horizontal advection terms are important in Equation 4.9. This is problematic because we know that entrainment affects vertical advection through MSE profiles, but we do not have a similar theory for horizontal advection. In order to reduce the role of horizontal advection in our analysis, we define a new GMS appropriate for the WC in our idealized simulations which we call the boundary GMS.

The boundary GMS is defined using MSE averaged over the boundaries of the
Figure 4-7: WC strength versus entrainment for a control climate (blue) and a warm climate (red).
WC ascent and descent regions which in our idealized simulations are defined as the elliptic ±10 W m$^{-2}$ surface Q flux contours. For each of the ascent and descent regions, we separate $h$ into an average over the elliptic boundary, $h_b$, and a residual, $h'$ such that $h = h_b + h'$. Considering the ascent region, the advection terms can now be written

$$\left\langle \frac{\partial h}{\partial p} \right\rangle_w + \langle \mathbf{u} \cdot \nabla h \rangle_w = \left\langle \frac{\partial h_b}{\partial p} \right\rangle_w + \langle \mathbf{u} \cdot \nabla h_b \rangle_w + \left\langle \frac{\partial h'}{\partial p} \right\rangle_w + \langle \mathbf{u} \cdot \nabla h' \rangle_w; \quad (4.10)$$

and similarly for the descent region. In order for $\left\langle \omega \frac{\partial h}{\partial p} \right\rangle_w$ to dominate the right-hand side, we need to define the boundary of the ascent region such that it has the following properties:

- In order for $\langle \mathbf{u} \cdot \nabla h_b \rangle_w$ to be negligible, $h_b$ cannot vary in the horizontal, only in the vertical
- In order for $\left\langle \frac{\partial h'}{\partial p} \right\rangle_w + \langle \mathbf{u} \cdot \nabla h' \rangle_w = \langle \nabla \cdot (\mathbf{u} h') \rangle_w$ to be negligible, by the divergence theorem, $h_b$ must be defined over a region with little horizontal advective flux of $h'$ through the surface.

Between the surface and top of atmosphere, the zonally-anomalous Q flux contours create an elliptic cylinder for each region. We define $h_b$ as the average value of $h$ around the elliptic contour at each level, so $h_b$ varies only in the vertical. We use $h_b$ around 10 W m$^{-2}$ for the ascent region, and average the resulting advection term over the area of the region, denoted $\left\langle \frac{\partial h_b}{\partial p} \right\rangle_w$. We use $h_b$ around -10 W m$^{-2}$ for the descent region, and average the resulting advection term over the area of the region, denoted $\left\langle \frac{\partial h_b}{\partial p} \right\rangle_e$. The difference between the two is denoted $\left\langle \frac{\partial h}{\partial p} \right\rangle_{w-e}$. This satisfies the two properties above to the extent that $h$ contours align with the -10 and 10 W m$^{-2}$ surface $Q*$ contours. The difference between the ascent and descent terms is denoted $\left\langle \frac{\partial h}{\partial p} \right\rangle_{w-e}$. Looking at Equation 4.10, we notice that since the two constraints are approximately satisfied by our definition of $h_b$, we have

$$\left\langle \frac{\partial h}{\partial p} \right\rangle_{w-e} + \langle \mathbf{u} \cdot \nabla h \rangle_{w-e} \approx \left\langle \frac{\partial h_b}{\partial p} \right\rangle_{w-e}. \quad (4.11)$$
We therefore define the boundary GMS as

$$\text{boundary GMS} = -g \frac{\langle \omega \partial h \rangle}{\omega_{w-c}}$$  \hspace{1cm} (4.12)

Looking at Figure 4-8 we can see that the relationship between WC response and boundary GMS response is much closer than the relationship between WC response and GMS. That for each simulation the boundary GMS response is closer to the negative of WC response indicates that the boundary GMS is a better metric than GMS for evaluating WC response across entrainment rates and climates.

### 4.3.6 GMS decomposition

Finally, we evaluate the response of WC to warming and compare it to the response of boundary GMS. Looking at Figure 4-9, we find that the responses of WC strength and boundary GMS are of opposite sign, consistent with the inverse relationship found in [Wills et al., 2017] and in the CMIP5 and AMIP models in Section 2. Further, both the weakening of the WC and the increase in GMS with warming dampen with entrainment. We decompose the response of boundary GMS to warming in the idealized simulation as we did in Section 4.2.5 using Equation 4.7 shown in Figures 4-10 and 4-11. Similar to the CMIP5 models, the $\Delta \omega$ contribution is larger than and of opposite sign from the $\Delta h$ contribution. The $\Delta h$ contribution is again the result of compensation between positive contributions due to temperature and geopotential height changes and a negative contribution from humidity changes. Again, the contribution from changes in humidity is dominated by changes in saturation specific humidity. The contribution from changes in temperature does not vary noticeably across entrainment rates when terms are in percent per K (Figure 4-10), which is surprising given the findings of [Held et al., 2007]. Nonetheless, the negative contribution from changes in $h$ profile increase with entrainment, though the increases appear to be dominated by changes in humidity. On the other hand, when changes are evaluated in units of GMS per K (Figure 4-11), the variations in the temperature contribution across entrainment rates is clear. The contribution from changes in tem-
Figure 4-8: Relationship between GMS response and WC response in idealized simulations. Delta indicates a perturbation from the $\dot{\epsilon} = 0$ and control climate simulation. Open circles indicate GMS response and closed circles indicate boundary GMS response. Black line is a reference line with slope of -1. Blue symbols indicate that the perturbed climate is a control climate and red symbols indicate that the perturbed climate is a warm climate.
Figure 4-9: Response of WC strength (blue) compared with response of boundary GMS (red) in idealized simulations with varying entrainment rates.

...temperature is weaker with increasing entrainment, consistent with a steeper lapse rate with increasing entrainment.

### 4.4 Discussion and conclusions

We have evaluated the response of the Walker circulation to warming, with an emphasis on the spread in the response across GCM projections. We use an MSE budget to do so, with the GMS response as a key parameter which is inversely proportional to the WC response. This is consistent with the heuristic idea that for a given energy transport, a higher GMS is associated with a weaker circulation. Further, we find a large spread of WC response to warming across CMIP5 models, with GMS response anticorrelated with WC response. We find a large of a spread in the AMIP models of...
Figure 4-10: Same as Figure 4-4 for idealized simulations with varying entrainment rates. GMS response has been replaced by boundary GMS response, consistent with Section 3.

Figure 4-11: Same as Figure 4-10 except changes are in units of J kg$^{-1}$ K$^{-1}$ instead of percent per K.
13.5% K\(^{-1}\), which can be compared to the CMIP5 models of 20.0% K\(^{-1}\), implicating the atmosphere as a key player in the response of the WC to warming. The strong role of the atmosphere does not preclude a strong role of the ocean since the spread from each component separately need not sum to the total spread of the coupled system. In addition, the ascent and descent regions of the WC are not in exactly the same location in each GCM. However, in the AMIP simulations the SST response is imposed and the same across GCMs, so the warm pool and cold tongue may not align with ascent and descent regions in all GCMs.

Nonetheless, we evaluate the role of the atmosphere, particularly the GMS, in the WC response to projected warming. GMS is set in part by MSE lapse rates. These lapse rates become steeper with entrainment, an uncertain and parameterized process in GCMs. Therefore, we evaluate the role of entrainment in setting GMS and WC strength in an idealized GCM. To do so, we modify the SBM convection scheme to represent entrainment, creating an entraining Betts-Miller scheme. We find that horizontal advection and their differences across simulations can be substantial, which is complicating because we do not have a theory for the relationship between entrainment and horizontal advection. To address this, we define a boundary GMS which approximately includes the role of horizontal advection while not involving horizontal velocities and MSE gradients. Rather, it represents vertical advection of MSE profiles averaged over the boundary of each of the ascent and descent regions. We find that the WC response weakens with warming, but more so at higher entrainment rates. The fractional changes in both WC strength and boundary GMS decrease in magnitude with increasing entrainment.

The role of radiation is smaller than that of GMS but nonetheless substantial in CMIP5 and AMIP models. The correlation between GMS response and WC response is -0.84 in the CMIP5 models and -0.79 in the AMIP models. The correlation between radiation response and WC response is 0.69 in the CMIP5 models and 0.78 in the AMIP models. We have focused on the role of GMS response, but it is clear that radiation response is also associated with WC response. Further, the radiation contribution is always the same sign in the GMS response; that is, contributing a
weakening in all but the inmcm4 model. We hypothesize that cloud and/or water-vapor radiative feedbacks are amplifying the WC and GMS responses in CMIP5 and AMIP models. The decomposition of WC responses in CMIP5 and AMIP models (Figures 4-1 and 4-2) indicate a strong role of WC-linked changes in radiation. Further, Peters and Bretherton (2005) found in a simple model that low clouds have a strong feedback on WC strength. Focusing on the descent region, low clouds tend to decrease with weakened circulation. These low clouds have a net cooling effect, so a reduction of low cloud in the descent region would tend to warm the region, further weakening the WC (Peters and Bretherton, 2005). Positive cloud and water-vapor radiative feedbacks would be consistent the smaller spread in WC and GMS responses in our idealized simulations, which lack cloud and water-vapor feedbacks.
Chapter 5

Conclusions and future research

5.1 Chapter 2: The response of tropical precipitation to warming

Chapter 2 provides a mechanism decomposition of the response of precipitation to warming over tropical oceans. We find that the wet-get-wetter mechanism robustly contributes increases in precipitation in wet regions of the control climate, which vary across models. However, the warmer-get-wetter mechanism does not contribute as substantial of changes as expected \textit{a priori} given its prominences in the literature. Instead, changes in surface convergence (SC) have a strong influence on the response of precipitation to warming, and these SC changes are related to the Laplacian of low-level warming. The relative contributions of the three mechanisms is qualitatively robust across models despite strong variations in precipitation response.

Despite the smaller-than-expected role of the warmer-get-wetter mechanism in our decomposition, we find that the sea surface temperature (SST) response acts via its influence on SC. More specifically, changes in the Laplacian of SST influence the response of SC via changes in pressure gradients. SST and Laplacian of SST are correlated, so the role of the Laplacian of SST in setting precipitation response does resemble a warmer-get-wetter mechanism to a small extent, albeit with substantial differences.
There are a number of areas of future work that stem from the findings of Chapter 2. Precipitation over land is more relevant to human habitability than precipitation over oceans. As an area of future work, a similar analysis can be applied over tropical land. In a preliminary analysis, the two-mode model becomes a one-mode model over land and different terms in the DSE budget dominate over land than over ocean. In particular, changes in surface sensible heat fluxes, which are negligible over ocean, are of opposite sign and are larger over land Muller and O’Gorman (2011). Finally, a preliminary analysis showed that vertical velocity may be related to surface albedo over land, so surface albedo could potentially replace SST when modifying the model for precipitation over land. The development of a DSE budget-based precipitation model over land has potential applications for understanding mechanisms of both control climate precipitation and the response to warming.

5.2 Chapter 3: Elucidating the gross moist stability

Chapter 3 derives an approximate expression for gross moist stability (GMS) as a function of SST and Laplacian of SST. This GMS approximation has a number of potential applications. For example, the quasi-equilibrium tropical circulation model (QTCM) of Neelin and Zeng (2000) includes a GMS parameter which does not take into account spatial variations in top heaviness of vertical velocity, \( \omega \). It is clear from Chapter 3 that spatial variations in \( \omega \) contribute substantially to spatial variations in GMS. Further, these spatial variations in top heaviness of \( \omega \) can be approximated as a function of SST and SC. Therefore, including our approximations for \( \omega \) has the potential to substantially improve the skill of the QTCM.
5.3 Chapter 4: The response of the Walker circulation to warming

Chapter 4 found a strong role of the atmosphere in setting both the response of the Walker circulation WC to warming and the spread in WC responses across CMIP5 and AMIP GCMs. In particular, the WC response is anticorrelated with GMS response across both CMIP5 and AMIP models. Further, the role of entrainment was evaluated in an idealized GCM. Variations in entrainment affect the moist static energy lapse rate, which affects the GMS. Variations in entrainment substantial enough to explain the spread in lapse rates in GCMs do not explain the spread in WC responses in CMIP5 and AMIP models. This suggests a substantial role of changes in the vertical shape of $\omega$ as it appears in the GMS. Investigating these changes is an important area of future work.

Due to the large role of radiation in our CMIP5 and AMIP analyses, and the lack of cloud and water-vapor radiative feedbacks in the idealized model, we hypothesize that cloud and water vapor-radiative effects may have a strong amplifying feedback which is contributing to the large spread across GCMs. The strong role of WC-linked radiation in Figures 4-1 and 4-2 supports this hypothesis. A possible way to test this hypothesis is to run an idealized GCM with the entraining Betts-Miller scheme modification introduced in Chapter 4 but include cloud and water-vapor radiative feedbacks. Further, we evaluated the role of entrainment because of its known effect on lapse rate, and we chose our entrainment values to have a similar spread in lapse rates to the spread across CMIP5 and AMIP models. However, GMS is also strongly affected by vertical velocity. Evaluating the spread in vertical velocity profiles across models may be a useful next step.
5.4 Discussion of relationships between chapters

5.4.1 Chapters 2 and 3

Chapters 2 and 3 are related in their methods; both use two-mode approximation and relate the shallow and deep modes to SC and SST. However, they address distinct scientific research questions. While Chapter 2 applies the two-mode approximations to $\omega$ as it appears in the DSE budget, Chapter 3 applies the two-mode approximations to $\omega$ as it appears in the GMS. Chapter 2 studies precipitation response to warming in CMIP5 models while Chapter 3 studies GMS in observations and reanalyses. In general, that the vertical velocity-SST/SC relationship holds when applied to both precipitation and GMS is not entirely surprising even though we did not use an MSE budget and GMS to study precipitation response in Chapter 2. Indeed precipitation is inversely proportional to GMS Raymond et al. (2009). Further, the results corroborate the need for two-modes of vertical-motion variability instead of one when considering geographic patterns of precipitation and the circulation over the tropics.

5.4.2 Chapters 2 and 3 vs. Chapter 4

The first and second chapters evaluate the role of SST and SST gradients in setting precipitation and circulation and finds that SST gradients have a strong influence on both precipitation response to warming and on GMS. However, in Chapter 4 the spread in WC responses across AMIP models is substantial and comparable to that of CMIP5 models despite no variations in SST and SST response. This is an apparent contradiction with the findings of Chapters 2 and 3. One possibility is that differences in the shape of vertical motion profiles across GCMs (that is, differences in $\dot{\omega}$) could explain the spread in WC responses. In addition, as seen in Chapters 2 and 3 there are errors in both the deep mode approximation and in the relationship between SC and Laplacian of SST. Further, the Chapter 2 supplement (Appendix B) finds that the two-mode model cannot adequately capture the portion of the weakening of the circulation due to spatially-uniform increases in SST because it does not adequately
capture the upward shift with warming Singh and O’Gorman (2012). Nonetheless, we find that the approximations are skillful in reproducing simulated precipitation responses. The relative roles of the shallow and deep modes may be different when estimating precipitation than when estimating WC strength. Quantifying the role of upward shifts on the spread in WC response across models is an area of future work that may help rectify the apparent discrepancy.
Appendix A

Further details of the two-mode model

Here we give additional details about the two-mode model and how it differs from the version of the model derived in BB09b.

Unlike BB09b, we use monthly climatological data throughout the paper, including for calculating the vertical motion profiles and as inputs to the two-mode model. Using monthly data rather than monthly climatological data would increase the RMSE values but would not affect our conclusions.

BB09b apply EOF analysis to convergence and integrate vertically to give modes in \( \omega \) whereas we apply EOF analysis directly to \( \omega \) for simplicity. The \( \omega \) values are linearly interpolated to an evenly-spaced pressure coordinate before the EOF analysis is applied, which eliminates the need for weighting by vertical grid spacing. The shallow and deep modes are linear combinations of these EOFs, chosen so that the shallow mode has zero near-surface convergence and the deep mode is orthogonal to the shallow mode. The shallow-mode structure is given by \( \Omega_s = -\Omega_1 + r \Omega_2 \) and the deep-mode structure is given by \( \Omega_d = r \Omega_1 + \Omega_2 \), where \( \Omega_1 \) is the first EOF and \( \Omega_2 \) is the second EOF. The ratio \( r \) is given by

\[
    r = \frac{-\Omega_2(1000\text{hPa}) - \Omega_2(950\text{hPa})}{\Omega_1(1000\text{hPa}) - \Omega_1(950\text{hPa})},
\]

(A.1)
Figure A-1: Vertical velocity profiles for the shallow (solid) and deep (dashed) modes from (a) ERA-Interim and (b) the ensemble-mean historical climate (blue) and the ensemble-mean future climate under RCP8.5 (red). Thin black lines in panel (a) represent the unrotated modes from ERA-Interim. The amplitude is arbitrary.

Figure A-2: Deep-mode amplitude $o_d$ calculated from climatological-mean monthly data over August 1999 through July 2009 and binned by observed climatological-mean monthly SST$_{rel}$ from NOAA OI SST and SC from QuikSCAT. Deep-mode amplitude is calculated from (a) ERA-Interim, (b) approximation used here (Eq. 2.6) with SST$_{rel}$ and SC as inputs, and (c) approximation used in BB09b (see Appendix A) with SST$_{rel}$ as input. Contour interval: 0.05 Pa s$^{-1}$. White contour bounds bins with more than 100 data points per bin. Heavy black contour corresponds to deep-mode amplitude of zero. In (b) and (c), values of $o_d$ where a Heaviside function is invoked, resulting in zero precipitation, are not shown.
where 1000hPa and 950hPa are the two lowest pressure levels. The mode structures are then normalized to have unit length.

The structures of the original EOFs and of the shallow and deep modes from ERA-Interim are shown in Fig. A-1. In ERA-Interim, the first and second EOFs explain 89% and 8% of vertical velocity variance, respectively, while the shallow-mode and deep-mode structures explain 70% and 27% of vertical velocity variance, respectively, with a total of 97% explained by the combination of the two modes. The ensemble-mean of the structures of the shallow and deep modes from the CMIP5 models are shown in Fig. A-1b. The mode amplitudes, $o_s$ and $o_d$, determine the overall magnitude of the vertical velocity, and are positive in regions of high SST$_{rel}$ and SC.

We also use a different approximation for the deep-mode amplitude $o_d$ as compared to BB09b. BB09b argue that the deep-mode amplitude is related to SST using column stability arguments and the weak temperature gradient approximation aloft, and they approximated the deep-mode amplitude as $o_d \approx a \text{SST} + b$, where $a$ and $b$ are regression coefficients. However, deep convection is best supported when there is boundary-layer convergence and lower-tropospheric moistening so that entrainment of dry air does not prevent deep convection. BB09b account for this by including a Heaviside function of SC in their precipitation model, effectively arguing that positive SC is a prerequisite for precipitation. The absence of precipitation for negative SC then implies a prediction for $o_d$ in order to close the DSE budget with no latent heating. The approximation used by BB09b for $o_d$ is calculated using SST from NOAA OI SST (Fig. A-2c) and is compared to the deep-mode amplitude as calculated using ERA-Interim reanalysis (Fig. A-2a). Regions where a Heaviside function is invoked such that precipitation is zero are not contoured in Figs. A-2b,c. The amplitudes are binned by SC and SST$_{rel}$. More attention should be given to the region inside the white contour in which bins have 100 or more data points. The approximation used by BB09b does not capture the negatively sloped contours of constant $o_d$.

Here we instead approximate $o_d$ as a linear regression with SST$_{rel}$ and SC as predictors (Eq. 2.6). The use of SST$_{rel}$ in lieu of SST makes the model more consistent with the warmer-get-wetter mechanism, and thus more appropriate for application to
climate change. The inclusion of SC in the deep-mode regression means that the deep-mode amplitude increases with shallow-mode amplitude consistent with the role of shallow moistening in favoring deep ascent. This new approximation does not require the Heaviside of SC to be included in the precipitation expression, but there is still a Heaviside function to prevent negative precipitation. The deep-mode amplitude approximated in this way agrees better with the deep-mode amplitude as calculated from reanalysis (Fig. A-2). The setting of precipitation to exactly zero by a Heaviside function for certain SC and SST\textsubscript{rel} values is a shortcoming of the $o_d$ approximations, but observed precipitation is light at those values (not shown).

Lastly, BB09b calculated their shallow-mode coefficient, $a_s$, using a linear regression whereas we calculate it using mass continuity, but similar values for $a_s$ emerge from both approaches.

Values for constants that appear in the two-mode model are given in Table S2.
Appendix B

Supplementary text: Further discussion of the role of weakening of the tropical circulation

Previous studies have found that the tropical divergent circulation weakens under projected climate change (e.g., Vecchi and Soden (2007)), and it is clear from the DSE budget (or moisture budget) that the resulting changes in vertical velocities should lead to a dynamic contribution to changes in precipitation. Part of the weakening of the circulation is related to the pattern of SST change He and Soden (2015) and this affects precipitation in our framework through the effect of both SST$_{rel}$ and $\nabla^2$SST on the shallow and deep modes. Indeed, a partial compensation between wetter and the contribution from changes in SC is evident in Fig 2-2. However, part of the weakening of the circulation relates to the spatial-mean component of the SST change and to increases in CO$_2$ Ma et al. (2012); He and Soden (2015); Merlis (2015) and these are not captured by our relations of the deep and shallow mode amplitudes to SST$_{rel}$ and $\nabla^2$SST, although they may be partly included in the two-mode model when SC(GCM) is used.

The AMIP_4K simulations help to quantify the importance of circulation weakening due to spatial-mean warming since these simulations have a spatially uniform increase in SST (Fig. SI). Circulation changes related to spatial-mean warming in
AMIP_4K may contribute to discrepancies between the GCM changes in precipitation and those from the two-mode model (e.g., in the central Pacific in Fig. S1a,b) and to the small changes in SC that occur despite a spatially uniform SST increase (Fig. S1). Nonetheless, the AMIP_4K precipitation response resembles the wet-get-wetter contribution and the difference between them does not resemble a weakening of the climatological precipitation pattern. Thus the contribution of a weakening circulation from spatial-mean warming does not seem to be a dominant contributor to the precipitation response. The weakening of the circulation is often measured by changes in $\omega$ at 500hPa, and we have confirmed that the two-mode model with unapproximated $o_s$ and $o_d$ captures a weakening of $\omega$ at 500hPa at a rate of 3.6% K$^{-1}$ as measured based on regression over ascent regions of the tropical oceans under AMIP_4K, though there is a large spread in the response that is not captured by the regression. Part of this weakening of $\omega$ at 500hPa comes from changes in the vertical structures of the modes (0.9 % K$^{-1}$) but most of it comes from changes in $o_s$ and $o_d$ (2.7 % K$^{-1}$). The contribution to $\Delta P$ from changes in $o_s$ and $o_d$ is a smaller weakening of 1.9 % K$^{-1}$ as measured by regression for AMIP_4K, but this contribution has both positive and negative values and again there is a large spread in the response that is not captured by the regression. This large spread explains why the difference between the total AMIP_4K precipitation response and the wet-get-wetter contribution does not resemble a weakening of the climatological precipitation (Fig. S1).

For RCP8.5 which includes both spatial-mean warming and changes in SST gradients, Chadwick et al. (2013) found that a dynamical weakening offset much of the wet-get-wetter (thermodynamic) component of the precipitation response in the tropics. They defined the thermodynamic component as the increase in precipitation at fixed convective mass flux, and this thermodynamic component scales with the low-level specific humidity at 7% K$^{-1}$. By contrast, we define wet get wetter as the contribution due to changes in $M_{ses}$ and $M_{sed}$ which gives a rate of increase of 5.6% K$^{-1}$ as measured by regression for AMIP_4K. Thus, less of a dynamical weakening

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1Most of this contribution comes from increases in $-\frac{\partial o_s}{\partial p}$ (5.1% K$^{-1}$) with smaller contributions
is needed in our formulation based on the DSE budget because of a smaller thermodynamic rate of increase as compared to the low-level specific humidity framework used by Chadwick et al. (2013). From changes in the vertical structure functions (0.9% K$^{-1}$) and from changes in radiation regression coefficients (-0.4% K$^{-1}$). The 5.1% K$^{-1}$ from changes in $\frac{\partial s}{\partial p}$ is different from the 3.5% K$^{-1}$ found in Muller and O’Gorman (2011) largely because Muller and O’Gorman (2011) normalize by global-mean surface warming whereas we normalize by surface warming over the tropical oceans.
Figure S1: As in Fig. 2-2 except for AMIP_4K simulation. Contour interval: 0.125 mm day$^{-1}$ K$^{-1}$. 
Figure S2: As in Fig. 2-2 except for AMIP_future simulation. Contour interval: 0.125 mm day$^{-1}$ K$^{-1}$. 
Table S1: List of GCMs used. Different subsets of GCMs are used in the two-mode model analysis in Section 2.3, in the MLM analysis in Sections 2.4 and 2.5, and in the AMIP analysis in Section 2.6, as indicated by the presence of an RMSE values. Values in the first column are the RMSE values for $\Delta P$ from the two-mode model for the RCP8.5 scenario (as compared to $\Delta P$ from the GCM). Values in the second column are RMSE values for $\Delta SC(\nabla^2 T_v)$ and $\Delta SC(\nabla^2 SST)$ for the RCP8.5 scenario, each as compared to $\Delta SC(GCM)$. Values in the third column are RMSE values for $\Delta P$ from the two-mode model for AMIP_pattern.

<table>
<thead>
<tr>
<th>Two-mode model</th>
<th>MLM</th>
<th>AMIP</th>
<th>AMIP_pattern</th>
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<td>RMSE $\Delta P$</td>
<td>RMSE $\Delta SC(\nabla^2 T_v)$, $\Delta SC(\nabla^2 SST)$</td>
<td>RMSE $\Delta P$ AMIP_pattern</td>
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<td>CanESM2</td>
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<td>NorESM1-M</td>
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Table S2: Two-mode model coefficients for ERA-Interim and for the ensemble mean of the two CMIP5 simulations and the three AMIP simulations used throughout the paper. The ERA-Interim version of the two-mode model uses NOAA optimal interpolation SST and QuikSCAT SC observations (see Section 2.2.3 of text). In the ERA-Interim version $\nabla^2 T_v$ and $\nabla^2 \text{SST}$ are spatially smoothed. First section: Radiation regression coefficients (Eq. 2.3). Second section: Two-mode model coefficients (Eq. 2.7). Third section: Deep-mode regression coefficients when SC($\nabla^2 T_v$) is used (Fig. 2-6d). Fourth section: Deep-mode regression coefficients when SC($\nabla^2 \text{SST}$) is used (Fig. 2-6e).

<table>
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<tr>
<th>Constant</th>
<th>ERA-I</th>
<th>CMIP5 hist</th>
<th>CMIP5 RCP8.5</th>
<th>AMIP_control</th>
<th>AMIP_4K</th>
<th>AMIP_future</th>
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<td>$b_{\text{SC}(\nabla^2 T_v)}$ (Pa)</td>
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Appendix C

The entraining simplified Betts-Miller convection scheme

The SBM convection scheme relaxes temperature profiles to a moist adiabat. Here, the scheme is modified such that temperature profiles are relaxed to an *entraining* adiabat. The entraining adiabat includes an entrainment rate, $\epsilon$, which varies inversely with height and is given by $\epsilon = \frac{\hat{\epsilon}}{z}$, where $\hat{\epsilon}$ is a unitless entrainment parameter and $z$ is height in meters. The convection scheme represents an ensemble of clouds, each of which detrains at a different level, which is crudely represented by the inverse relationship with $z$. Making use of the plume equation, which is valid above the lifting condensation level, the entraining adiabat is defined using

$$\frac{\partial h_s}{\partial z} = -\epsilon (h_s - h_e), \quad (C.1)$$

where $h_s = c_p T + g z + L r_s$ is the saturation MSE, $r_s$ is the saturation mixing ratio, and $h_e$ is the environmental MSE. Here we use the GCM’s gridbox MSE to represent the environmental MSE. Replacing $h_s$ on the LHS of Equation $C.1$ with the definition of $h_s$ and multiplying by $dz$ gives

$$c_p dT + gdz + L dr_s = -\epsilon (h_s - h_e) \, dz. \quad (C.2)$$
Using the fact that \( r_s = r_s(T, p) \) and applying the hydrostatic equation gives

\[
c_p dT + gdz + L \frac{\partial r_s}{\partial T} dT - L \rho g \frac{\partial r_s}{\partial p} dz = -\epsilon (h_s - h_e) dz. \tag{C.3}
\]

Next, we divide by \( \partial z \) and group like terms on the LHS to give

\[
(c_p + L \frac{\partial r_s}{\partial T}) \frac{\partial T}{\partial z} + g - L \rho g \frac{\partial r_s}{\partial p} = -\epsilon (h_s - h_e). \tag{C.4}
\]

Rearranging to solve for \( \frac{\partial T}{\partial z} \) gives

\[
\frac{\partial T}{\partial z} = \frac{-\epsilon (h_s - h_e) - g + L \rho g \frac{\partial r_s}{\partial p}}{c_p + L \frac{\partial r_s}{\partial T}}. \tag{C.5}
\]

The following expressions for the partial derivative of \( r_s \) with respect to pressure and with respect to temperature are derived from the approximation for the saturation mixing ratio and the ideal gas law:

\[
L \rho \frac{\partial r_s}{\partial p} = -\frac{L r_s}{RT} \tag{C.6}
\]

\[
L \frac{\partial r_s}{\partial T} = \frac{L^2 r_s}{R_v T^2}. \tag{C.6}
\]

These two expressions are substituted into Equation C.5 to give

\[
\frac{\partial T}{\partial z} = \frac{-\epsilon (h_s - h_e) - g(1 + \frac{L r_s}{RT})}{c_p + \frac{L^2 r_s}{R_v T^2}}. \tag{C.7}
\]

Using the hydrostatic equation and the ideal gas law, \( \partial z = -\frac{RT}{g} \partial \ln p \). Replacing \( \partial z \) in the denominator of the LHS, multiplying both sides by \( -\frac{RT}{g} \), and multiplying the numerator and denominator of the RHS by \( 1/c_p \) gives

\[
\frac{\partial T}{\partial \ln p} = \frac{\frac{RT}{g c_p} (h_s - h_e) + \frac{RT}{c_p} + \frac{L r_s}{c_p}}{1 + \frac{L^2 r_s}{c_p R_v T^2}}. \tag{C.8}
\]
Letting $b = \frac{L^2 c_p}{c_p R_e T^2}$ and using $\kappa = \frac{R}{c_p}$ gives our entraining moist adiabat

$$\frac{\partial T}{\partial \ln p} = \frac{\kappa T\left[\frac{\epsilon}{\gamma} (h_s - h_e) + 1\right] + \frac{L c_s}{c_p}}{1 + b}. \quad (C.9)$$

Notice that this entraining moist adiabat reduces to a moist adiabat when $\epsilon = 0$. 
Bibliography


